

# ADAPTIVE FORMATION CONTROL OF A FLEET OF UNDERWATER VEHICLE-MANIPULATOR SYSTEM

BY

SIDDIG MUSTAFA ELKHIDER ELNAIEM

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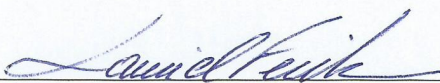
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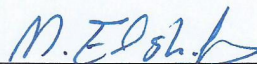
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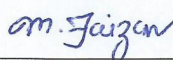
Thesis Committee



Dr. Sami El Ferik (Adviser)



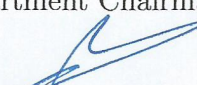
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Dr. M. Faizan Mysorewala (Member)



Dr. Fouad Al-Sunni  
Department Chairman



Dr. Salam A. Zummo  
Dean of Graduate Studies



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*To*  
*My Fathor , Mother, Brothers and Sisters*

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# LIST OF ABBREVIATIONS

$\eta$	Inertial Position of Vehicle
$\mathcal{L}_1$	One norm
$\nu$	Linear velocity in Body Fixed Frame
$\omega$	Angular velocity in Body Fixed Frame
$\tau$	Input Torques and Moments
$AUV$	Autonomous Underwater Vehicle
$C$	Coriolis and Centripetal Matrix
$D$	Drag Matrix
$DOF$	Degree of Freedom
$g$	Gravity and Buoyancy Forces Vector
$M$	Inertia matrix including Rigid Body and Added Mass
$MARES$	Modular Autonomous Robot for Environment Sampling
$R_B^I(\eta)$	Rotation Matrix
$UVMS$	Underwater Vehicle-Manipulator System

$x$	State of general dynamics
$y$	Output of general dynamics

# THESIS ABSTRACT

**NAME:** Siddig Mustafa Elkhider Elnaiem  
**TITLE OF STUDY:** Adaptive Formation Control Of a Fleet Of Underwater  
Vehicle-Manipulator System  
**MAJOR FIELD:** Systems Engineering  
**DATE OF DEGREE:** May 2014

In this thesis three problems are addressed. The first is the problem of a fleet formation under leader-follower strategy. The whole fleet navigation has been addressed using  $\mathcal{L}_1$  adaptive control and artificial potential field formation approach. The framework is developed to control the navigation and formation of a fleet of Underwater Vehicle-Manipulator System (UVMs). The proposed framework is developed by installing the potential fields on the group leader and, on the assumption that the leader is located in the group center, the potential fields formation control strategy is used to localize all followers around their leader. The formation control is developed such that the potential fields control inputs for the followers to control their desired positions around their leader. A new control technique,  $\mathcal{L}_1$  adaptive control is applied to general equations of motion of

the Underwater vehicle with a 3 link manipulator as inner controller. This  $\mathcal{L}_1$  adaptive controller has the ability to achieve fast and robust adaptation which increases the performance. The second is the problem of the containment control of the leader-followers with direct topology. This has been addressed using  $\mathcal{L}_1$  adaptive control and artificial potential field formation approach. By assuming that the leaders are located in square shape, the potential fields formation control strategy is used to localize all followers inside their leaders. The third is to study the effect of ocean disturbance on the formation control of a fleet of UVMs. Simulation results verify the high accuracy and robustness of  $\mathcal{L}_1$  adaptive control. The formation stability of each follower is analyzed using the Lyapunov method and its derivation is provided.



## ARABIC ABSTRACT

الاسم: صديق مصطفى الخضر النعيم .

عنوان البحث: تطبيق متحكم التشكيل التكيُفي على أسطول من الغواصات ذات الأذرع .

التخصص: هندسة النظم .

تاريخ البحث: مايو 2014 .

في هذا البحث تم معالجة ثلاثة مشاكل الأولى مشكلة تشكيل أسطول من الغواصات ذات الأذرع بإتباع إستراتيجية القائد والتابع . تم استخدام متحكم إل ون التكيُفي و نهج تشكيل الحقل المحتملة الاصطناعي لملاحه هذا الأسطول . فقد تم تطوير نظام للسيطرة والتحكم في تشكيل هذا الأسطول من الغواصات ذات الأذرع فقد تم تصميم النظام المقترح عن طريق تثبيت و نهج تشكيل الحقل المحتملة الاصطناعي على الغواصة التي تستخدم قائدا للأسطول المفترض أن تقع في مركز القائد للمجموعة، وقد تم تطوير نظام للتحكم في تموضع الغواصات ذات الأذرع في هذا الأسطول حول بعضها البعض ولمنع تصادمها بناء على استخدام خاصية مجالات الجذب والتباعد. تم تطبيق متحكم جديدة (إل ون التكيُفي) لأول مرة على الغواصات ذات الأذرع ذو الثلاثة مفاصل كمتحكمه داخلية . هذه المتحكمه تمتاز بالسرعة والمتانة ضد تغيرات البيئة المحيطة بالنظام مما يساعد على زيادة أداء النظام . المشكلة الثانية هي مشكلة احتواء مجموعة من القادة على أسطول من الغواصات ذات الأذرع في شكل شبكه ذات وصلات ثابتة . تم أيضا استخدام متحكمه إل ون التكيُفي و نهج تشكيل الحقل المحتملة الاصطناعي لمعالجة هذه المشكلة على افتراض أن القادة في شكل مربع . تم استخدام نهج تشكيل الحقل المحتملة الاصطناعي لتموضع الغواصات ذات الأذرع في شكل شبكة داخل أولئك القادة . المشكلة الثانية هي دراسة تأثير اضطرابات المحيط على تشكيل هذا الأسطول من الغواصات ذات الأذرع . النتائج أظهرت متانة وكفاءة عالية لمتحكمه إل ون التكيُفي . استقرار نظام التحكم المقترح تم تحليلها وإثباتها باستخدام طريقة ليابانوف .

## CHAPTER 1

# INTRODUCTION

High research effort has been focused on the underwater vehicle-manipulator systems (UVMS), which are widely used in underwater manipulation tasks including ocean exploration, oil and gas platform maintenance and underwater pipeline inspection. UVMS systems have two main parts. The first one is the Underwater Vehicle which is the complete mobile system by itself, and it has six degrees of freedom (6 DOFs) to control. The second part is the Underwater Manipulator which is a Robotic Arm characterized by  $n$  successive links, where,  $n$  represents the number of DOFs. In this work only revolute links are considered. When an Underwater Manipulator is attached to the Underwater Vehicle the total system is called Underwater Vehicle Manipulator System and the number of DOFs will become  $N = 6+n$  DOFs, as illustrated in Figure 1.1 . In recent years the multi-Autonomous Underwater Vehicle (AUVs) became more attractive for the researchers because it has more advantages than a single AUVs stemming from the ability to finish complex tasks more efficiently than a single AUVs. For ex-

ample, multiple AUVs can estimate their position faster and more accurately due to their ability to exchange information related to their positions whenever they sense each other . Many design strategies for controlling a group of AUVs have been formulated based on the required application. In many of these applications AUVs have to work cooperatively to integrate certain tasks or actions. In this study, a new framework for multiple cooperative manipulation tasks is proposed. One of the UVMs will be able to lead the others in an unknown environment. At the same time, all the UVMs in the group will keep the desired formation with each other and with their groups leader, and the whole system will be integrated to handle approach, organization and transportation control modes.

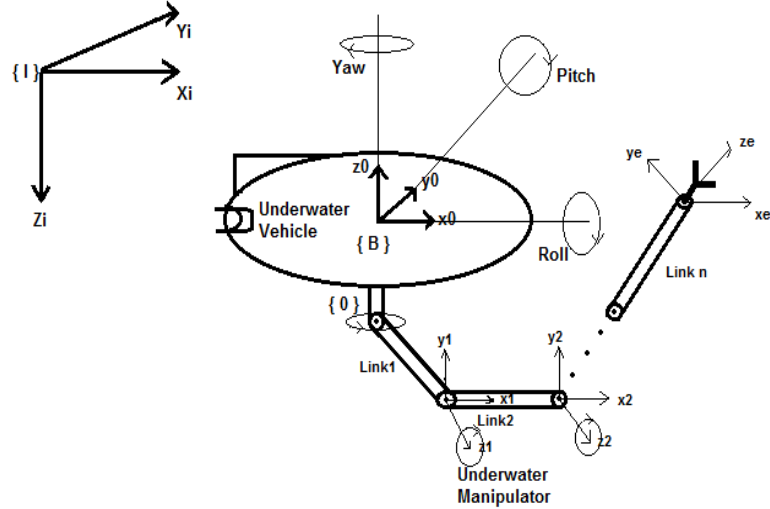


Figure 1.1: Underwater Vehicle Manipulator System.

## 1.1 Motivation

In recent years the multi-AUVs system has become more attractive to the interest of researchers because it has more advantages than a single AUVs system, such as the ability to finish complex tasks more efficiently than doing these tasks by using a single AUVs. For example, multiple AUVs can estimate their position faster and more accurately due to their ability in exchanging information related to their positions whenever they sense each other. Many design strategies for controlling a group of AUVs have been formulated based on the required application. In many applications the AUVs have to work cooperatively to integrate certain tasks or actions. For example, when a group of AUVs is used to explore an unknown environment the returned data from their sensors will be better than using a single AUVs for this task. A new framework for multiple cooperative manipulation tasks will be proposed. One of the UVMs will be able to lead the others in an unknown environment. At the same time, all the UVMs in the group will keep the desired formation with each other and with their group's leader, and the whole system will be integrated to handle approach, organization and transportation control modes. This framework will be developed by a combination and improvement in two stages. The first one is UVMs localization at each time step when the UVMs follows its trajectory. This process will be installed by running the potential fields on the group leader. The objective of this stage is to place an UVMs in an unknown location of an unknown environment and then the leader UVMs can incrementally build a map of this unknown environment, and at the same time use

this map simultaneously to navigate autonomously. The second stage is the group formation based on the artificial potential fields. The potential fields are a useful strategy for formation control consideration for a group of UVMS that allows one to control the behavior of a group of UVMS. Controlling the position of the UVMS with 9 DOFs poses some difficulties, most importantly, the disturbance (modeled in the dynamic equation) caused by the interaction forces between the AUVs and the manipulator when one of them changes its position.  $\mathcal{L}_1$  adaptive controller used for control design as the inner controller because  $\mathcal{L}_1$  adaptive controller has the ability to achieve fast and robust adaptation which increases the performance.

## 1.2 Thesis Objectives

The first objective of this thesis is to develop a new framework for handling cooperative tasks which will be built by integrating UVMS group formations based on the artificial potential fields combined with  $\mathcal{L}_1$  adaptive controller. The advantage of  $\mathcal{L}_1$  adaptive controller stems from its decoupling between robustness and adaptation leading to a robust control. In addition, the technique does not require exact knowledge of the non-linear dynamics. Simulation results and analytical analysis will be used to evaluate the performance.

This approach is described in the following:

1. This framework allows one of the UVMS to lead the others in an unknown environment, assuming that the leader configuration is used as the group field center. At the same time all the agents in the fleet will keep their formation

shape based on the potential fields. The cost function, will be used for analyzing the formation stability.

2. The whole system will be integrated to handle cooperative tasks.

The second objective is to develop a new framework containment control of the leader-followers with direct topology using  $\mathcal{L}_1$  adaptive control and artificial potential field formation approach.

Finally, the effect of ocean disturbance for the first objective is studied.

## 1.3 Thesis Organization

- Chapter 1 gives a brief introduction about the UVMS model supported by some motivations and real applications and the organization of the thesis.
- Chapter 2 consists of the literature about the formation control and the containment control of the leader-followers.
- Chapter 3 Modeling of Underwater Vehicle Manipulator System.
- Chapter 4 Controller Design for a Fleet Of Underwater Vehicle-Manipulator System.
- Chapter 5 The simulation results.
- Chapter 6 concludes the thesis.

## CHAPTER 2

# LITERATURE REVIEW

There are two main points in this section, the first one is the formation control of the leader-followers and the second point is the containment control of the leader-followers.

### 2.1 The formation control of the leader-followers

Alexandre Santos Brandao et al. [1] applied decentralized control on the kinematic model of mobile robots with Leader-follower . Decentralized control is used to drive the mobile robots to avoid the static obstacles. The leader robot seeking of goal to take care and avoidance of static obstacle, while the follower keeps the formation as a whole rigid body and identifies the leader's current pose with estimation of the linear velocity of the leader by a laser scanner. The leader-follower formation control is based on the inverse kinematics and the leader robot localizes the static environment and estimated its pose using laser scanner's pattern mounted on board the follower robot.



Yassine Bouteraa et al. [2] developed a multiple robot system of the leader-follower structure with backstepping control. A decentralized control law is designed for a team of fully actuated manipulators to follow a common desired trajectory and synchronize their movements. Synchronizing the velocities and positions of multiple followers interconnected via the neighbor-based role with respect to the state leader is based on the combination of graph theory and backstopping technique control.

Xingping Chen et al. [3] proposed a control formation of terrestrial UAVs as a leader-follower using decentralized control architecture. Dynamic inversion is used to estimate the unknown parameters of the controller. Unknown parameters of the reference trajectory and unknown parameters of the system model are handled and merged in an identical fashion. The formation control of the leader-follower of terrestrial UAVs is implemented by employing the information between its motion relative with it's leader.

K. Choi et al. [4] applied adaptive formation control of the kinematic model of mobile robots which is two-wheeled. The time-varying or time-invariant leader robot's velocities is estimated by the smooth projection algorithm. The follower robot used adaptive control to maintain the desired orientation, distance and bearing angle. This paper designed adaptive control law for each follower without the velocity information of the leader robot.

Stefana M. Cristescu et al. [5] applied a Cascade Control for Mobile Robots as Leader-Follower String Formation using a cascade PI-P controller. Each robot is a

leader for the next robot and a follower for the previous robot. No communication with Leader-Follower but the formation is implemented using distance measurements from image processing and speed measurements from optical encoders. A robust cascade PI-P controller is used for maintaining string formation, despite modeling errors and significant disturbances.

Saba Emrani et al. [6] proposed an adaptive formation control of leader-follower for multiple AUVs in spatial motions with desired trajectory and 3-dimensional spaces. Linear segments with parabolic blends (LSPB) method is used for generating suitable trajectories in 3-dimensional spaces of the environment, Adaptive inverse dynamics algorithm is used to make the leader robot to track this desired trajectory, the follower robot tries to maintain a desired angle and distance of the leader.

Jawhar Ghommam et al. [7] presented an obstacle avoidance based on Fuzzy Logic for Nonholonomic Robots Formation Control of Leader-Follower. The leader has defined its position measurement and the virtual vehicle is directed in a way that it stabilizes to the new shifted position or heading that was defined by the leader. The Lyapunov direct technique and backstepping control is designed to make the follower to track the virtual vehicle. Fuzzy logic control is designed to make both the leader and followers to avoid the inter-collision with others and the obstacles with a dynamic environment according to distance between obstacles and follower robots. The state leader defined the desired position of followers, by keeping orientation and distance of the leader the followers will reach the target

positions and the group will be established.

Adaptive formation control of Autonomous mobile robots with Leader-Follower is proposed by Jing Guo et al. [8]. In this paper autonomous mobile robots shaped as networks are controlled by an adaptive formation control. Only two leaders know the desired reference velocity and without the need to know the velocity of follower's neighbors. Each follower tried to maintain a specific triangular formation with two neighbors by adaptive formation control law. Two leader robots are required to save a desired distance between each other. All the followers follow its two neighbor robots and maintains a triangulation formation with them.

I. Kaminer et al. [9] applied L1 adaptive control on the kinematic model of multiple UAVs with time-critical missions and coordinated path following. Direct Method of calculus is used for generating trajectories for all UAVs in 3D space and all UAVs must track this path with the desired speed. Angular rate is augmented with an L1 adaptive output feedback control with guarantee stability and functioning of the organization. The communication network is used to replace the information between the vehicles. An inner-outer loop structure for vehicle coordination is achieved by L1 adaptive control.

Kishorekumar H Kowdiki et al. [10] applied artificial potential functions to leader-follower formation control with kinematic approach using an artificial potential field. An artificial potential field makes the leader robot to generate its navigation's path. It makes the leader robot follow an optimal path for reaching the goal position and avoid collisions with the static obstacles at the same time.

The followers tried to maintain the bearing angle and the separation distance and track the path by kinematic principles control.

Yang Li et al. [11] proposed flocking algorithm for Multi-Agent Control with Multi-leader Strategy. The Leaders choose their followers according to the capacity of followers accepted by leaders and the distances between the followers, the followers with the same leader are closed to others as a flocking during the tracking process. They know the velocity and position of each leader and match velocity with nearby flock mates . The followers with different leaders, on the other hand, are separated and do not have to match their velocity. The group of followers flock toward multiple leaders at the same time. This paper has some assumptions such as each leader can accept a certain number of followers, each follower has maximum velocity and it cannot exceed this given upper value. Leaders can communicate with each other and each follower knows the velocities and positions of its neighbor's leaders.

A formation control of group of mobile robots based on fuzzy logic as Leader-Follower is proposed by Marianne Sisto et al. [12]. A fuzzy logic controller is used for avoiding the internal collision between the followers and also for generating the desired trajectory. Each time the real leader sends its latest position to each robot, the robot calculates the position of its own virtual leader and follow this position.

Long Cheng et al. [13] proposed decentralized adaptive control of Leader-Follower with uncertain dynamics system. Backstepping scheme is designed as

a decentralized controller on the model of each followers manipulator. Adaptive control technique is used to deal with each follower manipulators to track the trajectory of leader.

Bo Liu et al. [14] proposed a potential field of Multi-Vehicle Systems, that, it makes the vehicles avoid collision with other and at the same time attract to each other. Using rules of nearest neighbor with fixed communication topology, the velocity and the direction of each vehicle controlled to achieve common values as well as under the communication topology switch. The follower robots depends on the weight's coupling and the leader's motion among the member vehicles.

LIU Shi-CAI et al. [15] applied robust control on second order kinematics model of mobile robots with leader-follower. Kinematics model of mobile robots is formulated relative to motion between the mobile robots. Feedback linearization is used as formation controller after linearizing the nonlinear model as well as applying sliding mode controller for stabilizing the internal dynamics and the overall system. Uncertainty parameters in the system is estimated by robust adaptive controller.

Jinyan Shao et al. [16] proposed a new framework for leader-following using distributed control and communication protocols. In this paper two controllers are applied to a dynamic model of leader-follower. The first controller is to help all following robot to maintain a desired pose of the leader and other controller to allow the follower robots to avoid the obstacles during its navigation. Each robot has a unique identification number (ID) to identify all others and to avoid

the collision with others.

Three-level hybrid architecture is developed: At the first level, virtual shell means the robot becomes the leader robot if it able to plan a proper path for the whole of the robots and it's able to exchange information with the other robots by broadcasting and receiving information. At the second level, the follower robots try to maintain a desired angle and desired distance relative to its leader. The third level, the leader robot is still the leader mod if it's able to plan a proper path, Otherwise, it will take the follower mode.

Touraj Soleymani et al. [17] applied Sliding Mode Control on the kinematic state-space model of autonomous air robots with leader following. Specific range and two angles determine the location of the follower with respect to the leader. The control of the leader following tried to maintain the relative distance, azimuth angle, and elevation angle of their desired values in a way that the vehicle track the leader in a desired direction determined by the leader's velocity. The control of vehicle system tried to stabilize the vehicle system and track the generated path. The equivalent control tried to eliminate the chattering phenomenon.

T. Sun , F. Liu et al. [18] proposed an adaptive control for a nonholonomic model of mobile robots with leader-follower technique. The follower doesn't know the dynamics of the leader and the follower robot estimate only the direction angle and the position of the leader robot by neural network control. Multiple sliding surface technique control is designed to make the followers robot to track the bearing angle and desired separation of the leader robot.

M. Naderi Soorki et al. [19] presented dynamic formation control of leader-follower with dynamic environments obstacle avoidance based on feedback linearization and sliding mode control. The follower robot tracks the path with the leader robot instead of saving the distance and relative bearing is based on transforming the formation control according to the relative motion between the leader robot and the follower robot to an equivalent tracking control for the follower robot. The backstepping control is applied to kinematic/torque for leader-follower while considering the avoidance of active obstacle. The follower robot has a desired distance to the obstacle and it needs to track the leader's path and also must track a new desired path when it senses an obstacle and it converts its desired trajectory to the obstacle, After that it must return to the primary desired path. The feedback linearization is applied to make the follower robot to track the leader robot with the desired relative bearing and desired distance. The sliding mode controller is designed to make the follower robot to estimate the position of the leader robot. The problem of active obstacle is solved by considering the obstacle as a virtual leader.

Yi Zhang et al. [20] proposed formation control of Multi-robot with Leader-follower and network. The real-time's problem solved by ad-hoc network, it makes the robots arrive at the goal position quickly and in correct lines. The followers should know themselves to avoid the collisions with others and the leader's position, that by maintaining the orientation to the leader and relative distance. Broadcasting mechanism is used to communicate the robots with others. Each



follower has the constant velocity of 1m/s and sensor for detecting obstacles and avoid it individually, and the follower begins to avoid the obstacle in the distance of 1m.

A classical control algorithm (backstepping control) is applied on the kinematic model of mobile robot with some communications technology during the followers is proposed by Zhuping Wang, Ying Mao et al. [21]. The communication between robots is achieved by cross-platform technology Qt and there isn't any interaction between robots. One robot has the information of the others by sensor and the communication doesn't depend on the environment, such that it must support the adaptation to the environment. The follower and leader robots communicate with other using client- server mechanism and will calculate the position of follower if the follower's angle and distance with respect to the leader and position leader are known, by calculating the bearing, relative separation, and other parameters . The controller information had been broadcasted from the server to the followers, that make them as clients, and after getting their own spatial information (their position), each one of these followers exchange the broadcasted information with their leader.

A collective of controller architecture for formation control of leader-follower robot based on fuzzy control, static environment and formation switching is proposed by Aleksandar Cosic et al. [22]. The leader track generated path designed by high level language while avoiding collision with an obstacle, The followers must be kept by the desired shape trajectory of the leader and track it, When it

senses an obstacle it must convert its desired trajectory to obstacle after that it must return to the primary desired path. Fuzzy logic controller and nonlinear IP controller is designed to avoid obstacles in a static environment and for tracking desired trajectory.

## 2.2 The containment control of the leader-followers

Yu Zhao et al. [23] proposed a new class of observer-based control algorithms for multi-agent systems based on distributed finite-time tracking control applied for second-order multi-agent, they proposed the containment control with two controllers, the first one was a multi-agent system with finite-time tracking protocol and multiple active leaders based on state feedback control, where the followers neighbors can receive the position states from others after the agent share its position, the second one was a multi-agent systems with finite-time tracking protocol and multiple active leaders based on output feedback control.

Ziyang Meng et al. [24] proposed a containment control for multiple rigid bodies based on Distributed finite-time attitude dynamics in lagrange expression with stationary leaders, the stationary leaders were converging by attitudes of the followers based on independent model control law. The sliding surface's non-singular and sliding-mode estimator are used when the system contains multiple dynamic leaders and this guarantees that the follower's angular velocities and followers's attitudes converge on dynamic leaders.

Jie Mei et al. [25] proposed a directed graph of networked lagrangian sys-

tems based on distributed containment control with uncertainties parametric and multiple dynamic leaders, an Euler-Lagrange equations of followers converge to dynamic leaders by combining two controllers, the first one is the sliding-mode estimators and the other is the adaptive control algorithm.

Ziyang Meng et al. [26] applied a containment control for multiple lagrangian systems and multiple stationary leaders with fractional-order powers in the control law. Independent model control law guaranteed that the follower's states converge to the stationary leaders. A non-singular sliding surface and sliding-mode estimator applied to the Multiple Lagrangian Systems with multiple dynamic leaders to sure that the dynamic leaders was converged by the followers in finite time.

Zhao-Jun Tang et al. [27] proposed a multiple stationary leaders with noisy measurements of multi agent systems based on containment control by consensus-like algorithm and stochastic, it presents that each agent converge with others in tree topology by the stationary leaders.

ZHAO Yu et al. [28] proposed a second-order of multi-agents systems with multiple dynamic leaders based on distributed finite-time containment control under direct topology with two types of controller dynamic output feedback and state feedback control under two cases, the first one when the finite-time consensus depend on the velocity states and relative position, the second case when the finite-time consensus depend on the relative position only.

Yu Zhao et al. [29] proposed a multiple dynamic leader with multi-agent in double-Integrator dynamic model and bounded unknown acceleration based on

distributed finite-time Containment Control. A containment controller was developed for multi-agent systems with velocity measurements and position measurements, also it was developed for multi-agent systems with position measurements only based on observer-type algorithm.

Di Yu et al. [30] proposed a nonlinear multi-agent networks with finite time containment control, switching control protocol and non-smooth continuous & discontinuous one based on sliding mode control. Sliding-mode proved that the followers converge the dynamic leaders in finite time.

Zhongkui Li et al. [31] Applied a multiple leaders of discrete-time and continuous-time multi-agent systems for general linear dynamics with directed topology based on Distributed containment control relative to the output of agents, the multi-step algorithm was applied to dynamic containment controller. If for each follower, at least there is one leader has a reference path to that follower then the follower's states can asymptotically converge to that leader.

Fang Yan et al. [32] proposed a time delay for multi-agent systems based on containment control, sampled-data-based control and adopting continuous-time, that both time delay and topology structure play an important role in the containment control of multi-agent systems with stationary/dynamic leaders with time-varying position and constant velocity.

Yongcan Cao et al. [33] proposed a distributed containment control for directed networks with multiple dynamic and stationary leaders under both switching and fixed directed network topologies was discussed for two cases, first, when all the

agents share an inertial coordinate frame and the leaders are stationary, second, when all the agents share an inertial coordinate frame and the leaders are dynamic. for second case without measuring for the velocity a distributed tracking control algorithm was proposed.

MEI Jie et al. [34] developed a neural network for multiple unknown second-order nonlinear systems based on containment control under a directed graph and external disturbances. a capability of neural networks was proposed with a distributed adaptive control algorithm.

Jianzhen Li et al. [35] presented a multiple dynamic leaders based on distributed containment control using only position measurements for double-integrator dynamics and multi autonomous vehicles model the followers can converge the dynamic leaders by Position Measurements only with undirected topology and for each follower, at least there is one leader has a reference path to that follower.

# CHAPTER 3

## MODELING OF

## UNDERWATER VEHICLE

## MANIPULATOR SYSTEM

The general kinematics and dynamics equations for Underwater Vehicle system are presented as follows.

### 3.1 Underwater Vehicle Kinematics Equation

In order to develop a UVMS model there are three reference frames which have to be considered. The first one is the earth fixed reference frame ( $\{I\}$  frame) and the second one is the body fixed reference frame ( $\{B\}$  frame) and last one is manipulator base reference frame located at the manipulator base ( $\{0\}$  frame).

Frame  $\{I\}$  is represented by  $\{I\}(O, x, y, z)$ , frame  $\{B\}$  is represented by  $\{B\}(O_B, x_B, y_B, z_B)$  and frame  $\{0\}$  is represented by  $\{0\}(O_0, x_0, y_0, z_0)$ , as shown in figure 1.1.

The position of the vehicle body frame with respect to the earth frame is denoted by the vector  $\eta_1 = [x \ y \ z]^\top$ , the orientation in angular position of vehicle body frame with respect to the earth frame is denoted by  $\eta_2 = [\psi \ \theta \ \phi]^\top$  which represent the yaw, pitch and roll, and the position of the manipulator angles with respect to the earth frame is denoted by the vector  $\eta_3 = [\theta_1 \ \theta_2 \ \dots \ \theta_n]^\top$ .

The translational and rotational movement of the UVMS with respect to the earth inertial frame can be described by the following vector:

$$\eta = [\eta_1 \ \eta_2 \ \eta_3]^\top = [x \ y \ z \ \psi \ \theta \ \phi \ \theta_1 \ \theta_2 \ \dots \ \theta_n]^\top \quad (3.1)$$

The velocity of the UVMS with respect to the earth inertial frame can be described by the following kinematic equation:

$$\dot{\eta} = R_B^I(\eta)\nu \quad (3.2)$$

Where  $\nu$  is the velocity of UVMS with respect to its frame(body frame), and it can be described by the following vector:

$\nu = [\nu_1 \ \nu_2 \ \nu_3]^\top$ ,  $\nu_1$  represent the linear velocity for the vehicle,  $\nu_2$  represent the angular velocity for the vehicle and  $\nu_3$  represent the joint velocity for the

manipulator links from link 1 to link  $n$ , thus,

$$\boldsymbol{\nu} = [\nu_1 \ \nu_2 \ \nu_3]^\top = [\nu_x \ \nu_y \ \nu_z \ \omega_x \ \omega_y \ \omega_z \ \dot{\theta}_1 \ \dot{\theta}_2 \ \dots \ \dot{\theta}_n]^\top \quad (3.3)$$

$\mathbf{R}_B^I(\eta)$  is the transformation matrix which transforms the velocities from body frame to the earth frame and it has the following components:

$$\mathbf{R}_B^I(\eta) = \begin{bmatrix} \mathbf{R}_{13 \times 3}(\eta) & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times n} \\ \mathbf{0}_{3 \times 3} & \mathbf{R}_{23 \times 3}(\eta) & \mathbf{0}_{3 \times n} \\ \mathbf{0}_{n \times 3} & \mathbf{0}_{n \times 3} & \mathbf{R}_{3n \times n}(\eta) \end{bmatrix} \quad (3.4)$$

Where  $\mathbf{R}_1(\eta)$  transforms  $\nu_1$  (linear velocity for the vehicle) to position rates in the earth frame and it has the following form:

$$\mathbf{R}_1(\eta) = \begin{bmatrix} C_\psi C_\theta & C_\phi S_\psi - C_\psi S_\phi S_\theta & -S_\phi S_\psi - C_\phi C_\psi S_\theta \\ C_\theta S_\psi & -C_\phi C_\psi - S_\phi S_\psi S_\theta & C_\psi S_\phi - C_\phi S_\psi S_\theta \\ -S_\theta & -C_\theta S_\phi & -C_\phi S_\theta \end{bmatrix} \quad (3.5)$$

Where  $C_x = \cos(x)$ , and  $S_x = \sin(x)$ .

$\mathbf{R}_2(\eta)$  transforms  $\nu_2$  (angular velocity for the vehicle) to the Euler rates in the earth frame and it has the following form:



$$\mathbf{R}_2(\eta) = \begin{bmatrix} 0 & -S_\phi/C_\theta & C_\phi/C_\theta \\ 0 & -C_\phi & S_\phi \\ 1 & -S_\phi S_\theta/C_\theta & -C_\phi S_\theta/C_\theta \end{bmatrix} \quad (3.6)$$

And  $\mathbf{R}_3(\eta)$  transforms the derivatives of the manipulator joint angles to the manipulator joint velocity in the earth frame and it has the following form:

$$\mathbf{R}_3(\eta) = \begin{bmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{bmatrix} = \mathbf{I}_{n \times n} \quad (3.7)$$

## 3.2 Underwater Vehicle Dynamics Equation

The equation of motion of UVMS system in the body fixed reference frame, located at the manipulator base can be written as the following :

$$\mathbf{M}(\eta)\dot{\nu} + \mathbf{C}(\eta, \nu)\nu + \mathbf{D}(\nu)\nu + \mathbf{g}(\eta) = \tau \quad (3.8)$$

Where

$$\mathbf{M}(\eta) = \begin{bmatrix} \mathbf{M}_v + \mathbf{H}(\eta) & \mathbf{M}_c(\eta) \\ \mathbf{M}_c^\top(\eta) & \mathbf{M}_m(\eta) \end{bmatrix}$$

Where  $\mathbf{M}_c(\eta)$  is the reaction inertia matrix between the mobile Vehicle and the manipulator,  $\mathbf{H}(\eta)$  is the added inertia due to the manipulator,  $\mathbf{M}_v$  is the inertia matrix for mobile vehicle and  $\mathbf{M}_m(\eta)$  is the inertia matrix for the manipulator.

The elements of the Coriolis and centripetal term ( $\mathbf{C}$  matrix) is computed by using the following expression:

$$C_{ij} = \frac{1}{2} \dot{M}_{ij}(\eta) + \frac{1}{2} \sum_{k=1}^n \left( \frac{\partial M_{ik}(\eta)}{\partial \eta_j} - \frac{\partial M_{jk}(\eta)}{\partial \eta_i} \right) \dot{\eta}_k$$

And the damping matrix as follow :

$$\mathbf{D}(\nu) = \begin{bmatrix} \mathbf{D}_v(\nu)_{6 \times 6} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_m(\nu)_{n \times n} \end{bmatrix}$$

Where  $\mathbf{D}_v(\nu)$  is the damping matrix for the mobile vehicle and  $\mathbf{D}_m(\nu)$  is the damping matrix for the manipulator.

$$\mathbf{g}(\eta) = \begin{bmatrix} \mathbf{g}_v(\eta) + \mathbf{g}_E(\eta) \\ \mathbf{g}_m(\eta) \end{bmatrix}$$

Where  $\mathbf{g}_v(\eta)$  is the gravity forces and moments vector for the vehicle,  $\mathbf{g}_E(\eta)$  is

the gravity forces and moments vector on the mobile vehicle due to manipulator and  $\mathbf{g}_m(\eta)$  is the gravity forces and moments vector for the manipulator.

$$\tau = \begin{bmatrix} \tau_v & \tau_m \end{bmatrix}^T$$

$\tau$  represents the external input forces vector, where  $\tau_v$  and  $\tau_m$  represents the input forces for the mobile vehicle and the input forces for the manipulator respectively.

Similarly the equation of motion for the UVMS in the earth fixed reference frame can therefore be written as the following:

$$\mathbf{M}_e(\eta)\ddot{\eta} + \mathbf{C}_e(\eta, \nu)\dot{\eta} + \mathbf{D}_e(\nu, \eta)\dot{\eta} + \mathbf{g}_e(\eta) = \tau_e \quad (3.9)$$

Where

$$\mathbf{M}_e(\eta) = \mathbf{R}^{-T}(\eta)\mathbf{M}(\eta)\mathbf{R}^{-1}(\eta)$$

$$\mathbf{C}_e(\eta, \nu) = \mathbf{R}^{-T}(\eta)[\mathbf{C}(\eta, \nu) - \mathbf{M}(\eta)\mathbf{R}^{-1}(\eta)\dot{\mathbf{R}}(\eta)]\mathbf{R}^{-1}(\eta)$$

$$\mathbf{D}_e(\nu, \eta) = \mathbf{R}^{-T}(\eta)\mathbf{D}(\nu)\mathbf{R}^{-1}(\eta)$$

$$\mathbf{g}_e(\eta) = \mathbf{R}^{-T}(\eta)\mathbf{g}(\eta)$$

$$\tau_e = \mathbf{R}^{-\mathbf{T}}(\eta)\tau$$

### 3.2.1 Properties of Equation of Motion

For the UVMS system the following properties hold:

- The inertia matrix for the total system is symmetric and positive definite

$$\mathbf{M}(\eta) = \mathbf{M}^{\mathbf{T}}(\eta) > \mathbf{0}$$

- The Damping matrix for the total system is strictly positive definite

$$\mathbf{D}(\eta, \nu) > \mathbf{0}$$

- Finally for the total system

$$\mathbf{x}^{\mathbf{T}} [\dot{\mathbf{M}}(\eta) - 2\mathbf{C}(\eta, \nu)] \mathbf{x} = \mathbf{0} \text{ is true}$$

where  $\mathbf{x}$  is an arbitrary vector.

We assume that the manipulator is attached at the vehicle centre, at a distance  $L/2$  forward centre of mass (C.M.) and  $H/2$  above mass centre. Thus, the position vector from the vehicle C.M. to the first joint of the manipulator is:

$$\mathbf{C}_0 = \begin{bmatrix} L/2 \\ 0 \\ H/2 \end{bmatrix} \text{ meters}$$

# CHAPTER 4

## CONTROLLER DESIGN FOR A FLEET OF UNDERWATER VEHICLE-MANIPULATOR SYSTEM

### 4.1 $\mathcal{L}_1$ Adaptive Control for the Underwater Vehicle

The attractive point of using  $\mathcal{L}_1$  adaptive controller in this thesis is its ability for fast and robust adaptation which increase the system's performance for both input and output compared to other controllers in the literature. This capability is done through the separation between adaption and robustness. Using this

controller the uncertainties will be estimated by a fast algorithm, therefore, compensated of these uncertainties will pass to a low-pass filter and it will be a part of the control signal. There are two main functions of this filter, first, guarantee that the control signal remains in the reasonable frequency range, second, separates between adaptation and robustness as well as through this filter we can optimize both the performance and the robustness (see[36,37,38]).

#### 4.1.1 Preliminaries

Given the nonlinear function  $f(t, x) : [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}$ , governed by the following assumptions.

- Assumption 1: (Uniform boundedness of  $f(t,0)$ ) There exists  $B > 0$ , such that  $|f(t, 0)| \leq B, \forall t \geq 0$
- Assumption 2: (Semiglobal uniform boundedness of partial derivatives)

If the nonlinear function  $f(t,x)$  is continuous in its arguments, and furthermore, for arbitrary  $\delta > 0$ , therefore, will exist  $d_{f_t}(\delta) > 0$  and  $d_{f_x}(\delta) > 0$ , such that  $\forall \|x\|_\infty \leq \delta$  the partial derivatives of  $f(t,x)$  with respect to  $t$  and  $x$  are piecewise continuous and bounded,

$$\left\| \frac{\partial f(t,x)}{\partial x} \right\|_1 \leq d_{f_x}(\delta), \text{ and } \left| \frac{\partial f(t,x)}{\partial t} \right| \leq d_{f_t}(\delta)$$

- Lemma

According to the above assumptions, the nonlinear function  $f(t,x)$  can be

linearly parameterized in two time-varying parameters using  $\|x\|_\infty$  as a regressor, as follows:

$$f(t, x(t)) = \theta(t)\|x(t)\|_\infty + \sigma(t)$$

Where,  $|\theta(t)| < \theta_\rho$ ,  $|\dot{\theta}(t)| < d_\theta$ ,  $|\sigma(t)| < \sigma_b$ , and  $|\dot{\sigma}(t)| < d_\sigma$

Where,  $\theta_\rho \triangleq d_{f_x}(\rho)$ ,  $\sigma_b \triangleq B + \epsilon$ , by which  $\epsilon > 0$  is an arbitrary constant,  $d_\theta$  and  $d_\sigma$  are computable bounds.

- Property 1: (see [39]) Given the vectors  $y \in \mathbb{R}^n$ ,  $\theta^* \in \Omega_0 \subset \Omega_1 \subset \mathbb{R}^n$ , and  $\theta \in \Omega_1$ , therefore :

$$(\theta - \theta^*)^\top (\text{proj}(\theta, y) - y) \leq 0 \quad (4.1)$$

#### 4.1.2 $\mathcal{L}_1$ Adaptive Controller Formulation

In this section  $\mathcal{L}_1$  adaptive controller components illustrated in figure 4.1 are presented. To apply  $\mathcal{L}_1$  adaptive controller on the UVMs, we have to rewrite (3.9) to be in the form of the state space representation. First, we will consider the following simple example [36,40]:

$$\begin{aligned} \dot{x}_1(t) &= x_2, \quad x_1(0) = x_{1_0} \\ \dot{x}_2(t) &= A_2 x_2(t) + f_2(t, x(t)) + B_2 w u, \quad x_2(0) = x_{2_0} \\ y(t) &= C x(t) \end{aligned} \quad (4.2)$$

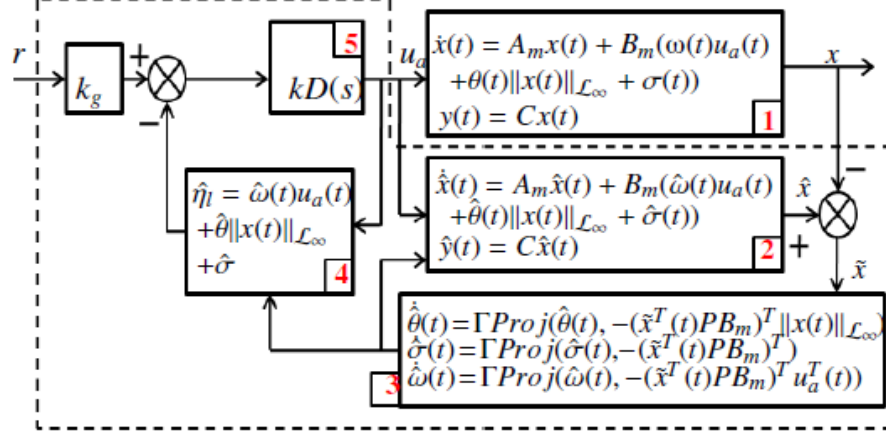


Figure 4.1: Block Diagram of the closed-loop  $\mathcal{L}_1$  adaptive controller [37].

Where,  $x(t) = [x_1(t), x_2(t)]^\top \in \mathbb{R}^{2n}$  are the states of the systems,  $A_2 \in \mathbb{R}^{n \times n}$  is a known matrix,  $B_2 \in \mathbb{R}^{m \times n}$  is a constant full rank matrix,  $u(t) \in \mathbb{R}^m$  is the control input vector,  $w \in \mathbb{R}^{m \times m}$  is the uncertainty on the input gain,  $C \in \mathbb{R}^{m \times n}$  is a known full rank constant matrix,  $y(t) \in \mathbb{R}^m$  is the measured output and  $f_2(t)$  is an unknown nonlinear function. We can rewrite 4.2 as:

$$\dot{x} = Ax(t) + f(t) + B_m w u(t) \quad (4.3)$$

Where,

$$A = \begin{bmatrix} 0_{n \times n} & I_{n \times n} \\ 0_{n \times n} & A_2 \end{bmatrix}, \quad f = \begin{bmatrix} 0_{n \times 1} \\ f_2 \end{bmatrix} \quad \text{and,} \quad B_m = \begin{bmatrix} 0_{n \times m} \\ B_2 \end{bmatrix} \quad (4.4)$$

From Eq. 3.9

$$\ddot{\eta} = M_e(\eta)^{-1} [\tau_e - C_e(\eta, \nu) \dot{\eta} - D_e(\nu, \eta) \dot{\eta} - g_e(\eta)] \quad (4.5)$$



Say:

$$\eta_1 = \eta$$

$$\eta_2 = \dot{\eta}, \quad \text{thus}$$

$$\dot{\eta}_1 = \dot{\eta} = \eta_2$$

$$\dot{\eta}_2 = \ddot{\eta} \tag{4.6}$$

Therefore, from Eqs. 4.5 and 4.6 the state space representation is:

$$\begin{aligned} \begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} &= \begin{bmatrix} 0_{6 \times 6} & I_{6 \times 6} \\ 0_{6 \times 6} & \frac{-D_e}{M_e} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} - \begin{bmatrix} 0_{6 \times 1} \\ \frac{g_e}{M_e} \end{bmatrix} + \begin{bmatrix} 0_{6 \times 6} \\ \frac{1}{M_e} \end{bmatrix} \tau \\ y(t) &= \begin{bmatrix} I_{6 \times 6} & 0_{6 \times 6} \\ 0_{6 \times 6} & I_{6 \times 6} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \end{aligned} \tag{4.7}$$

Now, we can rewrite Eq. 4.7 to be in the same form as Eq. 4.3 which leads to the Eq. 4.8.

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + B_m (w(t) u_{ad} + \theta(t) \|x(t)\|_\infty + \sigma(t)), x(0) = x_o \\ y(t) &= c^T x(t) \end{aligned} \tag{4.8}$$

Thus, the error dynamics equation can be obtained as follows:

$$\tilde{x}(t) = A_m \tilde{x}(t) + B_m (\tilde{w}(t) u_{ad} + \tilde{\theta}(t) \|x(t)\|_\infty + \tilde{\sigma}) \tag{4.9}$$

Where,  $\tilde{x}(t) = x(t) - \hat{x}(t)$ ,  $\tilde{w}(t) = w(t) - \hat{w}(t)$ ,  $\tilde{\theta}(t) = \theta(t) - \hat{\theta}(t)$  and  $\tilde{\sigma}(t) = \sigma(t) - \hat{\sigma}(t)$ .

Our systems is asymptotically stable when we assumed the projection operator as the following:

$$\begin{aligned}\dot{\tilde{\theta}} &= \gamma_1 Proj(\tilde{\theta}(t), -\|x(t)\| B_m^\top P \tilde{x}(t)) \\ \dot{\tilde{\sigma}} &= \gamma_2 Proj(\tilde{\sigma}(t), -B_m^\top P \tilde{x}(t)) \\ \dot{\tilde{w}} &= \gamma_3 Proj(\tilde{w}(t), -x^\top(t) P B_m u_{ad})\end{aligned}\tag{4.10}$$

where,  $(\gamma_1, \gamma_2 \text{ and } \gamma_3) > 0$  are the adaptation laws rate,  $\mathbf{P} = \mathbf{P}^\top > 0$  satisfy the Lyapunov equation;

$$\mathbf{A}_m^\top \mathbf{P} + \mathbf{P} \mathbf{A}_m = -\mathbf{Q}\tag{4.11}$$

where,  $\mathbf{Q} = \mathbf{Q}^\top > 0$ .

## 4.2 Formation control based on potential fields approach

In recent years, the potential field became a useful technique for formation control of a group of robots. There are many application used a group of robots to finish complex tasks. For example, multiple UVMS can estimate their position faster and more accurately due to their ability to exchange information related to their

positions whenever they sense each other. The potential fields approach can be applied to a fleet of robots moving in a three dimensional space or two dimensional space. The main idea of potential field technique is making a fleet of robots move in a formation of a desired trajectory.

#### 4.2.1 Shape formation

We assume that each robot has a sensing range equal to  $S_r$  figure (4.2), and each robot can know its neighbors positions which are located inside its sensing area. All nonholonomic robots in the fleet need to navigate in a desired polygon shape. These robots localize themselves as followers around a moving leader. The distance between each two neighboring followers should be equal to  $L \leq S_r$ . The radius of the circumcircle of the desired polygon is equal to  $r$ . The coordinates of the moving leader is  $x_c \in R^2$  which can be generated by potential field approach on the group leader. From the basics of the geometry,  $r$  will be: [42].

$$r = \frac{L}{2 \sin(\pi/n)} \quad (4.12)$$

where  $n$  is number of agents.

The following dynamics describe a fleet of 'n' point mass holonomic robots considered by formation control technique : [42]

$$\ddot{x}_i = u_i \quad i = 0, 1, 2, \dots \quad (4.13)$$

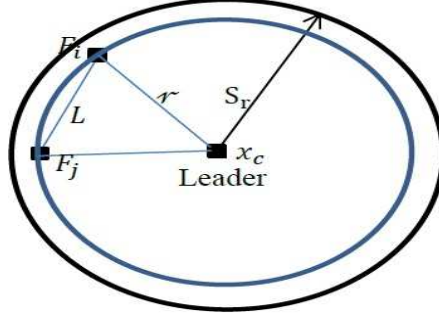


Figure 4.2: The leader sensing range  $F_i, F_j$  followers.

$x_i \in R^2$  is the  $i^{th}$  robot's position. If the robots are needed to formation themselves on the circumcircle of a regular polygon, the radius of the circumcircle of the polygon is  $r$ , and the distance between each two neighboring robots is equal to  $L$ .

We can obtain the control law of desired formation as follow :

$$u_i = f_{ci} + \sum_{j=1}^n f_{aij} - b\dot{x}_i \quad (4.14)$$

Where

$$f_{ci} = -\nabla_{x_i} V_{ci}(x_i) \quad (4.15)$$

where  $f_{ci}$  represent the force between the leader (center) and the follower  $i$ . This term keep all the followers around the leader with distance  $r$ .

$$f_{aij} = -\nabla_{x_i} V_{aij}(x_i, x_j) \quad (4.16)$$

where  $f_{aij}$  represent the force between the follower i and the follower j. This term repulses all the followers from other with distance L.

The control action in (4.14) was null, when all agents became at a distance r from the leader. Also the control action in (4.15) is null, when the distances between each two followers became equal to L.

We considered our formation shape as a regular polygon. For stablizing the system choose all energy of the system as a Lyapunov function :

$$\tilde{V} = \sum_{i=1}^n [Pot_i + Kin_i] \quad (4.17)$$

Where the kinetic equation :

$$Kin_i = \sum_{i=1}^n \frac{1}{2} V_i^T M V_i \quad (4.18)$$

And the potential equation :

$$Pot_i = P_{ci} + P_{aij} \quad (4.19)$$

The system will be stable if and only if  $\tilde{V}(p_i) > 0$ , and the time derivative of a Lyapunov candidate function  $\dot{\tilde{V}}(p_i) < 0$  :

$$\dot{\tilde{V}} = \sum_{i=1}^n [\nabla Pot_i] \dot{p}_i + \sum_{i=1}^n \frac{\partial}{\partial v_i} [\frac{1}{2} V_i^T M V_i] \dot{V}_i \quad (4.20)$$

The analysis of formation control stability will be discussed at the end of this

chapter.

### 4.2.2 Potential Field Formulation

The attractive, repulsive potentials and the vehicle damping action will be implemented as the control law. This control law will be obtained by the desired behavior of the system :

$$u_i = P_{ci} + P_{aij} + Da \quad (4.21)$$

where :

$P_{ci}$  The center potential.

$P_{aij}$  The interagent potential (agent's potential).

$Da$  Damping action.

### 4.2.3 Holonomic model

The holonomic robot model behaving like a point mass, the complete analysis of potential field formation can be expressed as follows.[42]

## Center Potential

The center potential between the follower  $i$  and the leader in  $R^3$  is defined as follows:

$$P_{att} = -\nabla_{x_i} V_{c_i}(x_{f_i}) \quad (4.22)$$

where

$$V_{c_i} = \frac{1}{2} K_c (d_{c_i} - r)^2 \quad (4.23)$$

where  $d_{c_i} = ||x_{f_i} - x_c||$  : is the Euclidian distance between the follower  $i$  and the leader.

$K_c$  : positive constant.

The control action between the leader and the follower  $i$  is as follows :

$$\mathbf{x}_{f_i} = \begin{bmatrix} \mathbf{x}_f \\ \mathbf{y}_f \\ \mathbf{z}_f \end{bmatrix} \quad \mathbf{x}_c = \begin{bmatrix} \mathbf{x}_l \\ \mathbf{y}_l \\ \mathbf{z}_l \end{bmatrix}$$

$$d_{c_i} = \sqrt{(x_f - x_l)^2 + (y_f - y_l)^2 + (z_f - z_l)^2} \quad (4.24)$$

$$\implies V_{c_i} = \frac{1}{2} K_c (d_{c_i} - r)^2 \quad (4.25)$$

By taking the differentiate of  $V_{c_i}$  with respect to  $x_{f_i}$ , we will obtain the center potential between the follower i and the leader. This differentiation will be as follows:

$$P_{tt} = -\nabla_{x_{f_i}} V_{c_i}(x_{f_i}) = -\frac{\partial V_{c_i}(x_{f_i})}{\partial x_{f_i}} \quad (4.26)$$

By using chain rule we can obtain:

$$-\frac{\partial V_{c_i}(x_{f_i})}{\partial x_{f_i}} = -\frac{\partial V_{c_i}}{\partial d_{c_i}} \frac{\partial d_{c_i}}{\partial x_{f_i}} \quad (4.27)$$

From equation (4.24)

$$\frac{\partial V_{c_i}}{\partial d_{c_i}} = K_c(d_{c_i} - r) \quad (4.28)$$

define

$$d_{c_i} = D^{\frac{1}{2}} \quad (4.29)$$

then from equation (4.23)

$$D = (x_f - x_l)^2 + (y_f - y_l)^2 + (z_f - z_l)^2 \quad (4.30)$$



by using the chain rule and differentiate  $d_{c_i}$  with respect to  $x_{f_i}$  we obtain:

$$\frac{\partial d_{c_i}}{\partial x_{f_i}} = \frac{\partial d_{c_i}}{\partial D} \frac{\partial D}{\partial x_{f_i}} \quad (4.31)$$

where

$$\frac{\partial d_{c_i}}{\partial D} = \frac{1}{2} D^{-\frac{1}{2}} \quad (4.32)$$

and

$$\frac{\partial D}{\partial x_{f_i}} = \frac{\partial D}{\partial x_f} + \frac{\partial D}{\partial y_f} + \frac{\partial D}{\partial z_f} = 2(x_f - x_l) + 2(y_f - y_l) + 2(z_f - z_l) \quad (4.33)$$

then

$$\frac{\partial D}{\partial x_{f_i}} = 2(x_{f_i} - x_c)^T \quad (4.34)$$

Then substitute (4.31) and (4.33) in (4.30) we will get:

$$\frac{\partial d_{c_i}}{\partial x_{f_i}} = \frac{1}{2} D^{-\frac{1}{2}} 2(x_{f_i} - x_c)^T \quad (4.35)$$

that means

$$\frac{\partial d_{c_i}}{\partial x_{f_i}} = \frac{1}{d_{c_i}} (x_{f_i} - x_c)^T \quad (4.36)$$

Finally substitute (4.35) and (4.27) in (4.26). The center potential between the follower i and the leader is:

$$P_{c_e} = \frac{-K_c}{d_{c_i}}(d_{c_i} - r)(x_{f_i} - x_c)^T \quad (4.37)$$

### **Repulsive Potential:**

The repulsive potential between the follower i and the follower j in  $R^3$  is defined as follows:

$$P_{rep} = -\nabla_{x_{f_i}} V_{a_{ij}}(x_{f_i}, x_{f_j}) \quad (4.38)$$

Where

$$V_{a_{ij}} = \begin{cases} \frac{1}{2}K_a(d_{ij} - L)^2 & d_{ij} < L \\ 0 & \text{Otherwise} \end{cases} \quad (4.39)$$

where  $d_{ij} = ||x_{f_i} - x_{f_j}||$ : is the Euclidian distance between the follower i and the follower j .

$K_a$  : positive constant.

The control action between the follower i and the follower j is as follows:

$$\mathbf{x}_{f_i} = \begin{bmatrix} \mathbf{x}_{f_i} \\ \mathbf{y}_{f_i} \\ \mathbf{z}_{f_i} \end{bmatrix} \quad \mathbf{x}_{f_j} = \begin{bmatrix} \mathbf{x}_{f_j} \\ \mathbf{y}_{f_j} \\ \mathbf{z}_{f_j} \end{bmatrix}$$

$$d_{ij} = \sqrt{(x_{f_i} - x_{f_j})^2 + (y_{f_i} - y_{f_j})^2 + (z_{f_i} - z_{f_j})^2} \quad (4.40)$$

then the repulsive potential between the follower i and the follower j is :

$$P_{rep} = \frac{-K_a}{d_{ij}}[d_{ij} - L][(x_{f_i} - x_{f_j})^T + (y_{f_i} - y_{f_j})^T] \quad (4.41)$$

#### 4.2.4 Nonholonomic model

This section will give the complete analysis of potential field formation for such a system has potential analysis which includes  $(x,y,z,\theta)$  instead of  $(x,y,z)$  as expressed in previous section, where  $(x,y,z)$  are the coordinates of the robot's center of the mass in the inertial Cartesian frame and  $(x,y,z,\theta)$  is the orientation of the robot with respect to the inertial frame.

### Center Potential :

The center potential for nonholonomic mobile robot is as follows :

$$P_{att} = -\nabla_{p_i} V_{c_i}(p_i) \quad (4.42)$$

and

$$V_{c_i} = \frac{1}{2} K_c (R_{c_i} - r)^2 \quad (4.43)$$

where  $R_{c_i} = ||x_{f_i} - x_c||$ : is the Euclidian distance between the follower i and the leader.

$$\mathbf{p_i} = \begin{bmatrix} \mathbf{x_i} \\ \mathbf{y_i} \\ \mathbf{z_i} \\ \theta_i \end{bmatrix} \quad \mathbf{p_c} = \begin{bmatrix} \mathbf{x_c} \\ \mathbf{y_c} \\ \mathbf{z_c} \\ \theta_c \end{bmatrix}$$

That means

$$R_{c_i} = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2 + (\theta_i - \theta_c)^2} \quad (4.44)$$

$K_c$  : positive constant.

By taking the differentiate of  $V_{c_i}$  with respect to  $p_i$ , we will obtain the center

potential between the follower  $i$  and the center robot. This differentiation will be as follows:

$$P_{att} = -\nabla_{p_i} V_{c_i}(p_i) = -\left(\frac{\partial V_{c_i}}{\partial R_{c_i}}\right)\left(\frac{\partial R_{c_i}}{\partial p_i}\right) \quad (4.45)$$

by differentiation (4.42) we will get:

$$\frac{\partial V_{c_i}}{\partial R_{c_i}} = K_c(R_{c_i} - r) \quad (4.46)$$

if

$$D = (x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2 + (\theta_i - \theta_c)^2 \quad (4.47)$$

Then

$$R_{c_i} = D^{\frac{1}{2}} \quad (4.48)$$

then by using the chain rule and differentiate  $R_{c_i}$  with respect to  $p_i$  we obtain:

$$\frac{\partial R_{c_i}}{\partial p_i} = \frac{\partial R_{c_i}}{\partial D} \frac{\partial D}{\partial p_i} \quad (4.49)$$

where from (4.47)

$$\frac{\partial R_{c_i}}{\partial D} = \frac{1}{2} D^{-\frac{1}{2}} \quad (4.50)$$

and

$$\frac{\partial D}{\partial p_i} = \frac{\partial D}{\partial x_i} + \frac{\partial D}{\partial y_i} + \frac{\partial D}{\partial z_i} + \frac{\partial D}{\partial \theta_i} = 2(x_i - x_c) + 2(y_i - y_c) + 2(z_i - z_c) + 2(\theta_i - \theta_c)$$

$$\frac{\partial D}{\partial p_i} = 2(p_i - p_c)^T \quad (4.51)$$

By substituting (4.49) and (4.50) in (4.48) we will get:

$$\frac{\partial R_{c_i}}{\partial p_i} = \frac{1}{2} D^{-\frac{1}{2}} 2(p_i - p_c)^T \quad (4.52)$$

then

$$\frac{\partial R_{c_i}}{\partial p_i} = \frac{1}{R_{c_i}} (p_i - p_c)^T \quad (4.53)$$

then By substituting (4.52) and (4.45) in (4.44) the center potential between the follower i and the center robot is :

$$P_{c_e} = \frac{-K_c}{R_{c_i}} (R_{c_i} - r)(p_i - p_c)^T \quad (4.54)$$

### Repulsive Potential :

The repulsive potential between each two neighboring followers in  $R^3$  can be expressed as:

$$P_{ij} = -\nabla_{p_i} V_{ij}(p_i, p_j) \quad (4.55)$$

Where

$$V_{ij} = \begin{cases} \frac{1}{2} K_a (R_{ij} - L)^2 & R_{f_i} < L \\ 0 & \text{Otherwise} \end{cases} \quad (4.56)$$

where  $R_{f_i} = ||p_i - p_j||$  : is the Euclidian distance between each two neighboring followers .

$K_a$  : positive constant.

The control action between each two neighboring followers :

$$\mathbf{p_i} = \begin{bmatrix} \mathbf{x_i} \\ \mathbf{y_i} \\ \mathbf{z_i} \\ \theta_i \end{bmatrix} \quad \mathbf{p_j} = \begin{bmatrix} \mathbf{x_j} \\ \mathbf{y_j} \\ \mathbf{z_j} \\ \theta_j \end{bmatrix}$$

$$R_{f_i} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 + (\theta_i - \theta_j)^2} \quad (4.57)$$

then the repulsive potential between the follower i and the follower j is :

$$P_{ij} = \frac{-K_a}{R_{f_i}}(R_{f_i} - L)((p_i - p_j)^T + (p_j - p_i)^T) \quad (4.58)$$

### 4.3 Stability Analysis

To get the desired formation we consider our stable configuration to be a regular polygon shape. Choose the total energy of the closed loop system as a Lyapunov candidate function  $\tilde{V}$

$$\tilde{V} = \sum_{i=1}^n [Pot_i + Kin_i] \quad (4.59)$$

Where the kinetic equation :

$$Kin_i = \sum_{i=1}^n \frac{1}{2} V_i^T M V_i \quad (4.60)$$

The system will be stable if and only if  $\tilde{V}(p_i) > 0$ , and the time derivative of a Lyapunov candidate function  $\dot{\tilde{V}}(p_i) < 0$ :

$$\dot{\tilde{V}} = \sum_{i=1}^n [\nabla Pot_i] \dot{p}_i + \sum_{i=1}^n \frac{\partial}{\partial v_i} [\frac{1}{2} V_i^T M V_i] \dot{V}_i \quad (4.61)$$

$$\dot{\tilde{V}} = \sum_{i=1}^n [\nabla Pot_i] \dot{p}_i + \sum_{i=1}^n V_i^T M \dot{V}_i \quad (4.62)$$



We know that

$$\dot{p}_i = \dot{\eta} = R_B^I(\eta)\nu_i \quad (4.63)$$

And

$$\dot{V}_i = A_m V_i + B_m u_i \quad (4.64)$$

By substituting (4.63) and (4.62) in (4.61) and by taking the transpose we will get:

$$\dot{\tilde{V}} = \sum_{i=1}^n [\nabla Pot_i] R V_i + \sum_{i=1}^n V_i^T M [A_m V_i + B_m u_i] \quad (4.65)$$

$$\dot{\tilde{V}} = \sum_{i=1}^n V_i^T R^T [\nabla Pot_i]^T + \sum_{i=1}^n V_i^T M [A_m V_i + B_m u_i] \quad (4.66)$$

For the desired configuration the system must be stable. For that we need to choose the control as:

$$u_i = M^{-1} (B_m B_m^T)^{-1} B_m [-R^T [\nabla Pot_i]^T - M A_m V_i - K_v V_i] \quad (4.67)$$

$$u_i = M^{-1} B_m^\dagger [-R^T [\nabla Pot_i]^T - M A_m V_i - K_v V_i] \quad (4.68)$$

Now substitute (4.67) in (4.65) we will get:

$$\dot{\tilde{V}} = \sum_{i=1}^n V_i^T R^T [\nabla Pot_i]^T + \sum_{i=1}^n V_i^T M [A_m V_i + B_m [M^{-1} B_m^\dagger [-R^T [\nabla Pot_i]^T - M A_m V_i - K_v V_i]]] \quad (4.69)$$

Then from Eq. 4.68 we can get

$$\dot{\tilde{V}} = \sum_{i=1}^n R^T V_i^T [\nabla Pot_i]^T - \sum_{i=1}^n R^T V_i^T [\nabla Pot_i]^T + \sum_{i=1}^n V_i^T M A_m V_i - \sum_{i=1}^n V_i^T M A_m V_i - \sum_{i=1}^n V_i^T K_v V_i \quad (4.70)$$

Finally from Eq. 4.69 we can get

$$\dot{\tilde{V}} = - \sum_{i=1}^n V_i^T K_v V_i < 0 \quad (4.71)$$

Which proves stability .

## CHAPTER 5

# SIMULATION RESULTS

In this thesis , The Modular Autonomous Robot for Environment Sampling (MARES)

AUV is used for leader-followers. The parameters of the MARES model as follows:

Properties	Value
Length	1.5 m
Diameter	20 cm
Weight in air	32 kg
Depth rating	100 m
Propulsion	2 horizontal + 2 vertical thrusters
Horizontal velocity	0-1.5 m/s, variable
Energy	Li-Ion batteries, 600Wh
Autonomy/Range	about 10 hrs / 40 km

Table 5.1: MARES General Characteristic

Properties	Value by m	Description
$[x_{cg}, y_{cg}, z_{cg}]$	$[0 \ 0 \ 0]$	Center of gravity
$[x_{cb}, y_{cb}, z_{cb}]$	$[0 \ 0 \ 4.4.10^{-3}]$	Center of buoyancy

Table 5.2: MARES Location of center of gravity and buoyancy

Properties	Value	Unit
$X_{\dot{u}}$	-1.74	kg
$Y_{\dot{v}}$	4.28.10	kg
$Z_{\dot{w}}$	-4.12.10	kg
$K_{\dot{p}}$	$-8.61.10^{-3}$	$kg.m^2$
$M_{\dot{q}}$	-6.07	$kg.m^2$
$N_{\dot{r}}$	-6.40	$kg.m^2$
$X_{\dot{q}}$	$-3.05.10^{-2}$	kg.m
$Y_{\dot{p}}$	$3.05.10^{-2}$	kg.m
$K_{\dot{v}}$	$3.05.10^{-2}$	kg.m
$M_{\dot{u}}$	$-3.05.10^{-2}$	kg.m
$Y_{\dot{r}}$	$1.13.10^{-1}$	kg.m
$Z_{\dot{q}}$	$-1.23.10^{-1}$	kg.m
$M_{\dot{w}}$	$-1.23.10^{-1}$	kg.m
$N_{\dot{v}}$	$1.13.10^{-1}$	kg.m

Table 5.3: MARES Added Mass

Properties	Value [ $kg.m^2$ ]
$I_{xx}$	$1.55.10^{-1}$
$I_{yy}$	4.73
$I_{zz}$	4.73

Table 5.4: MARES Moment Inertia

The above designed  $\mathcal{L}_1$  adaptive controller and potential field are implemented, with the parameters of the controller selected as (controller parameters).

$\gamma_1=10^4$ ,  $\gamma_2=10^4$ ,  $\gamma_3=10^4$ ,  $k=10^3$  and distance between each two followers  $L=2$ .

## 5.1 Results of leader-follower formation in 2D

### 5.1.1 Formation of UVMs without disturbance in input

The simulation results of the leader-follower group formation and whole fleet navigation are shown in the figures below. The figures from figure (5.1) to figure (5.3) are shown how the group of three UVMs can localize themselves around their leader in 2D space.

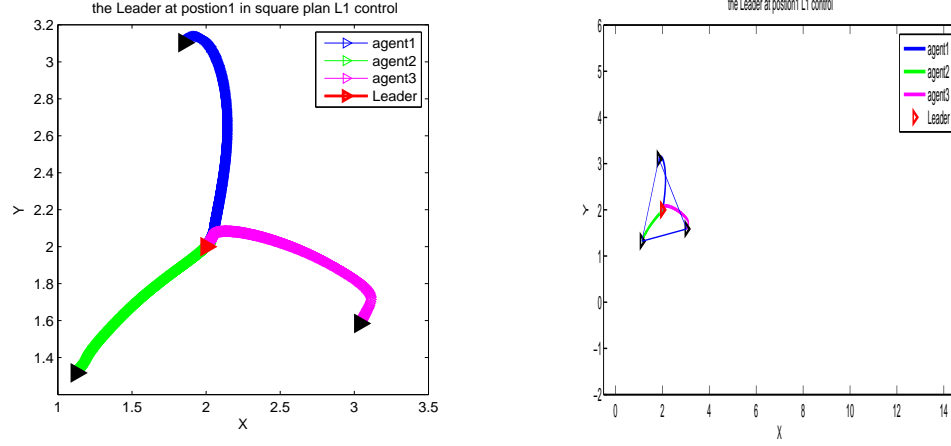


Figure 5.1: Leader and followers at position (2,2).

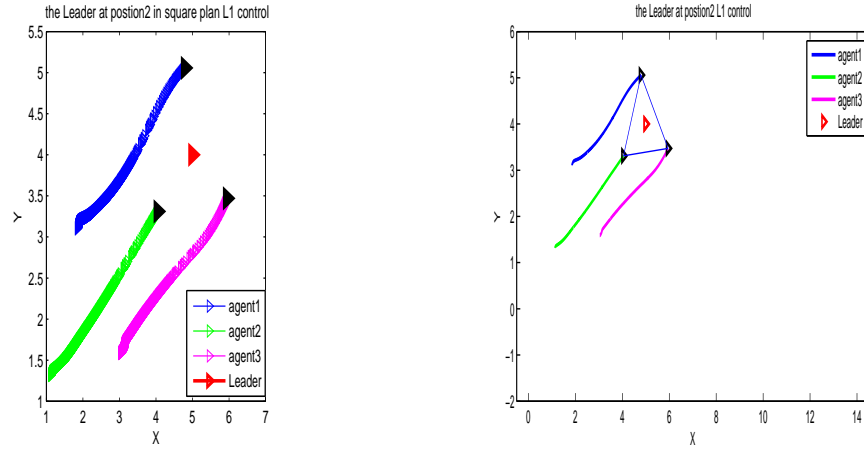


Figure 5.2: Leader at (5,4) and followers at final positions in figure 5.1 .

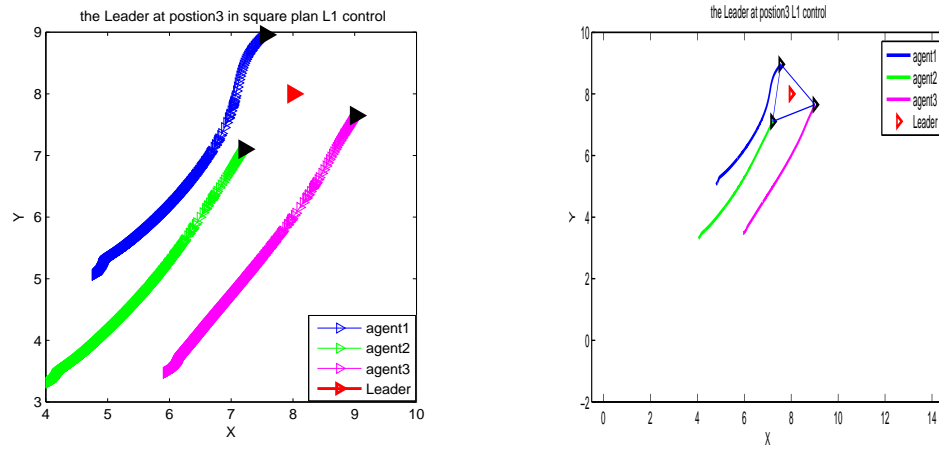


Figure 5.3: Leader at (8,8) and followers at final positions in figure 5.2.

### 5.1.2 Formation of UVMs with disturbance in input

The disturbance in input was not considered in the above results, but now it is considered for two scenarios, when the disturbance is gaussian noise (mean=1,frequency=10 and duty variance=0.3) and the second scenario the disturbance is square wave (amplitude=1,frequency=1000 and duty cycle=50%), the figures from figure (5.4) to figure (5.9) show that. The map of a fleet of three agents along sine shape with their leader under formation control based on L1 Adaptive control is shown in figure (5.10)

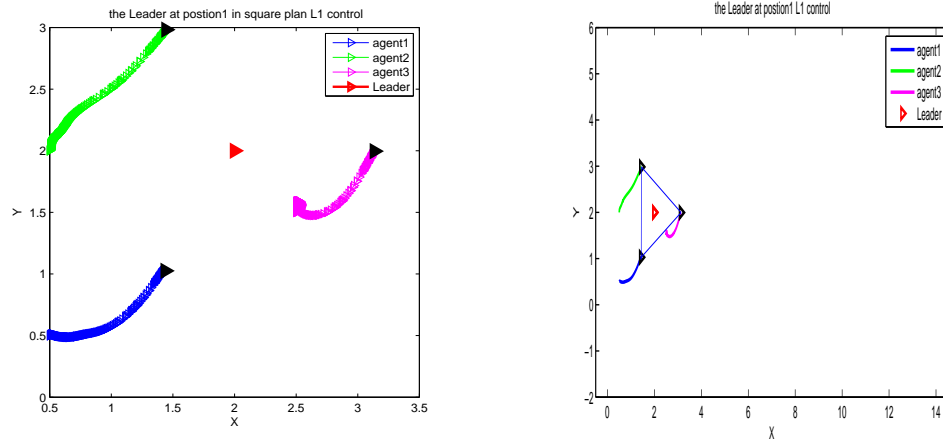


Figure 5.4: Leader and followers at position (2,2) with gaussian noise .

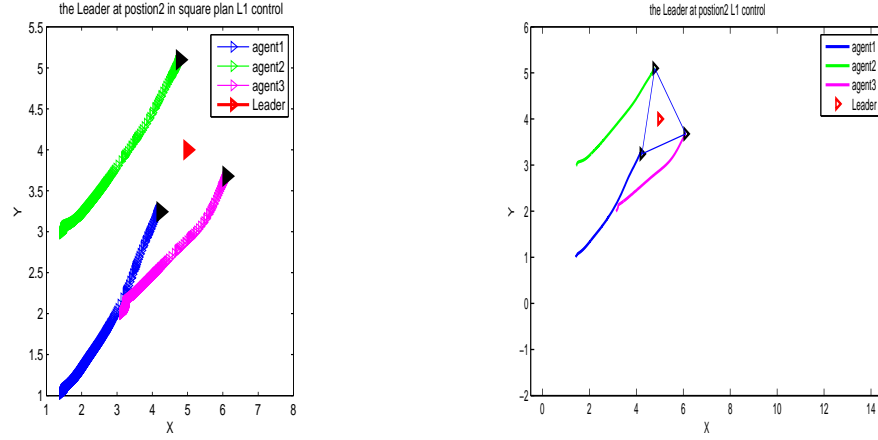


Figure 5.5: Leader at (5,4) and followers at final position in figure 5.4 with gaussian noise.

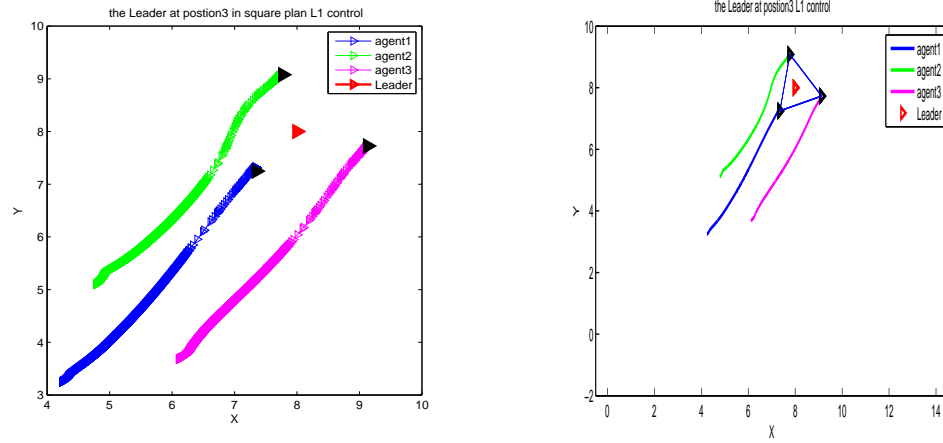


Figure 5.6: Leader at (8,8) and followers at final position in figure 5.5 with gaussian noise .



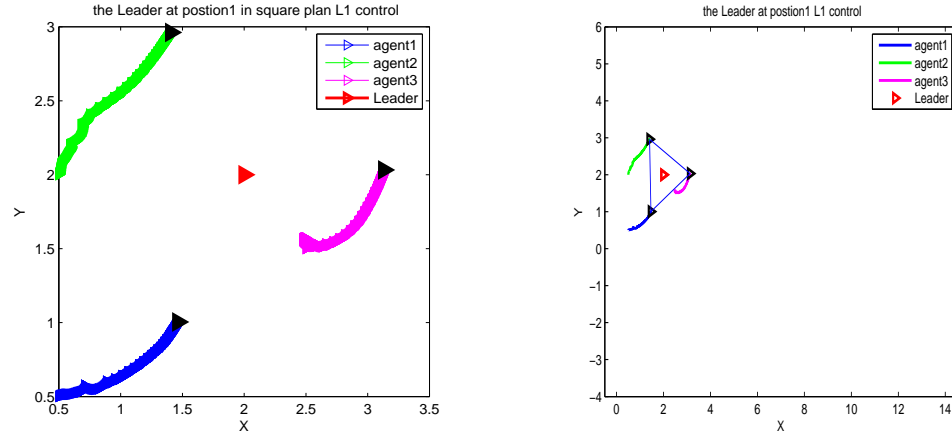


Figure 5.7: Leader and followers at position (2,2) with square wave .

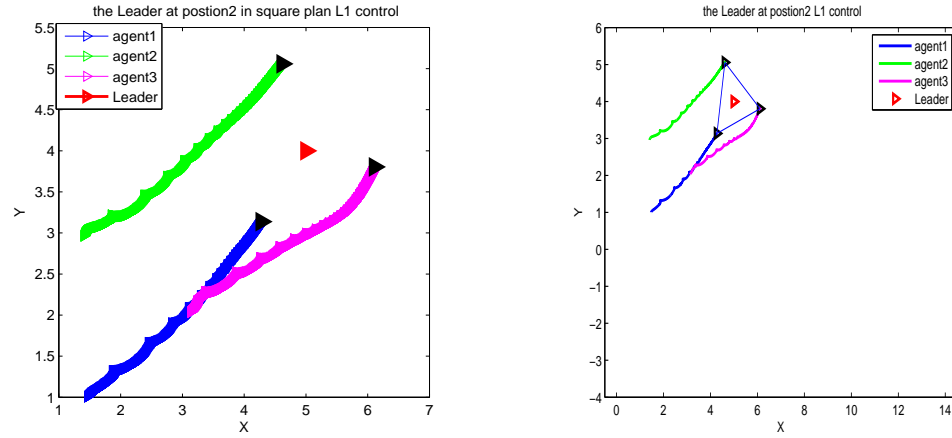


Figure 5.8: Leader at (5,4) and followers at final positions in figure 5.7 with square wave .

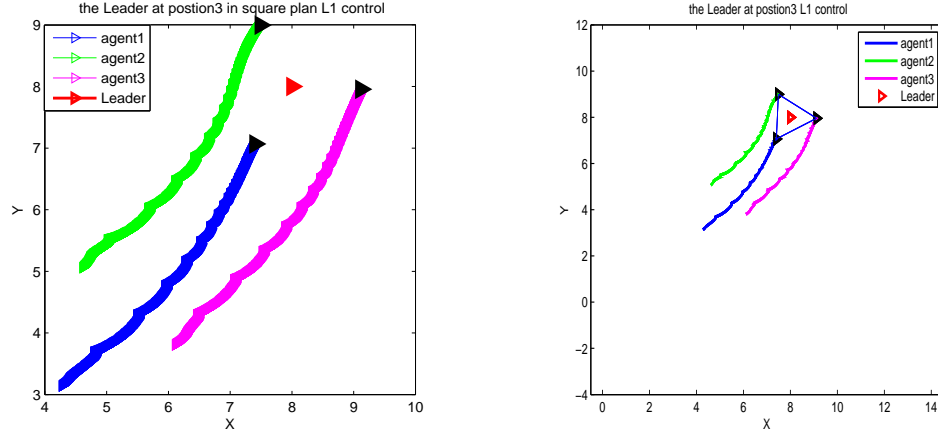


Figure 5.9: Leader at (8,8) and followers at final positions in figure 5.27 with square wave .

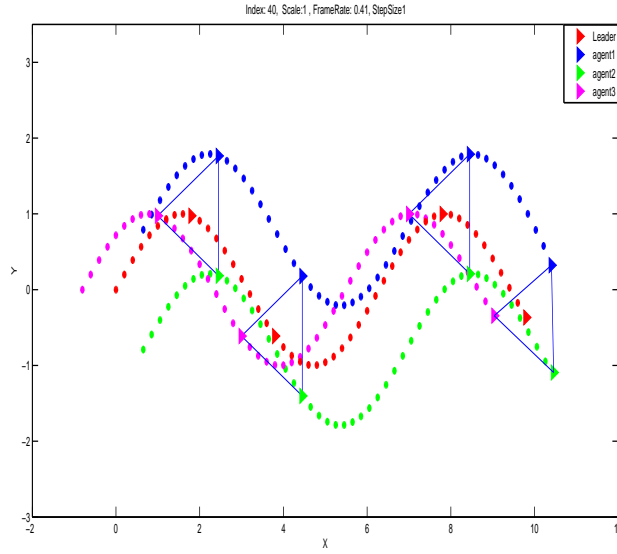


Figure 5.10: The map of a fleet of three agents along sine shape . with their leader under formation control based on  $L_1$  Adaptive control .

## 5.2 Results of leader-follower formation in 3D

In this section two different cases have been considered:

### 5.2.1 Vehicle doing task

In this case the manipulator remained fixed while the vehicle doing task, and this considered with/without uncertainty in parameters.

#### 5.2.1.1 Formation of UVMs without uncertainty in parameters

The figures from figure (5.11) to figure (5.14) are shown how the group of three UVMs can follow their leader in 3D space.

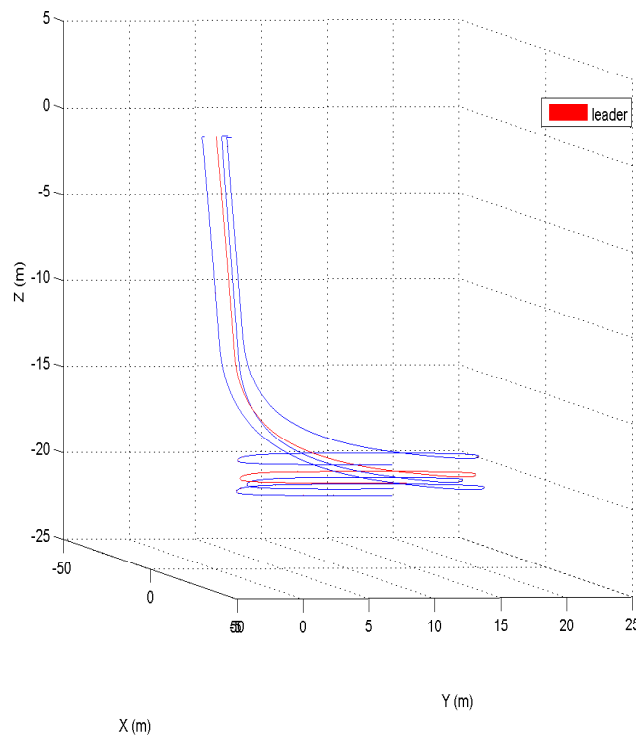


Figure 5.11: 3D trajectory tracking of the leader-follower without uncertainty.

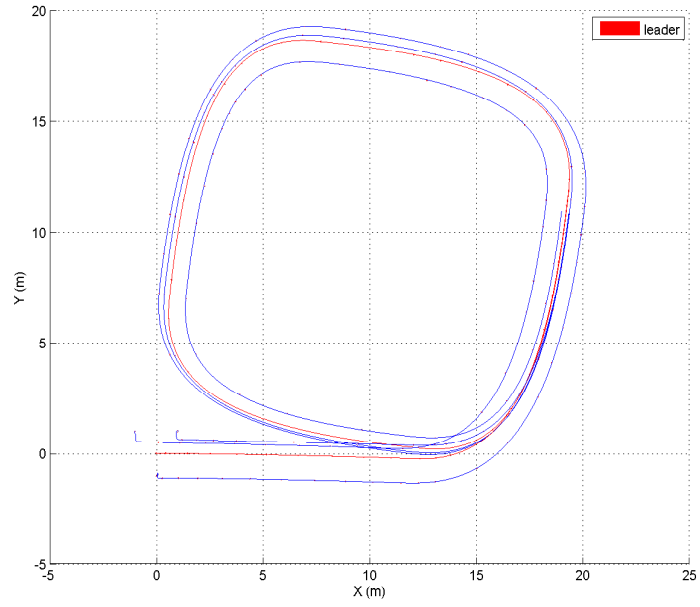


Figure 5.12: X-Y Semi Circle trajectory tracking of the leader-follower without uncertainty in parameters.

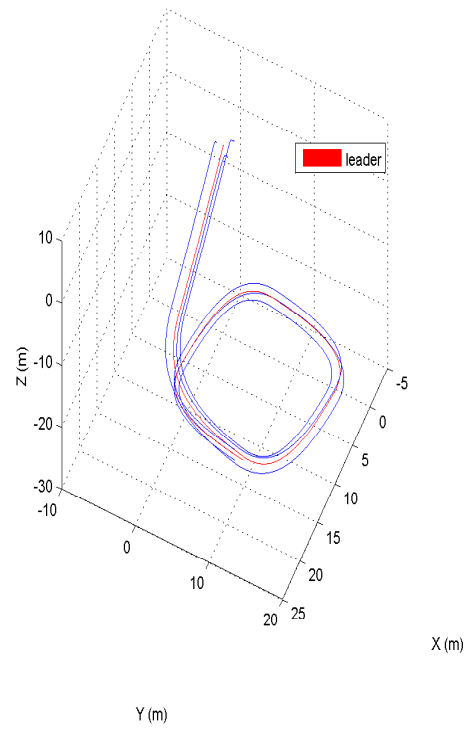


Figure 5.13: 3D trajectory tracking of the leader-follower without uncertainty.

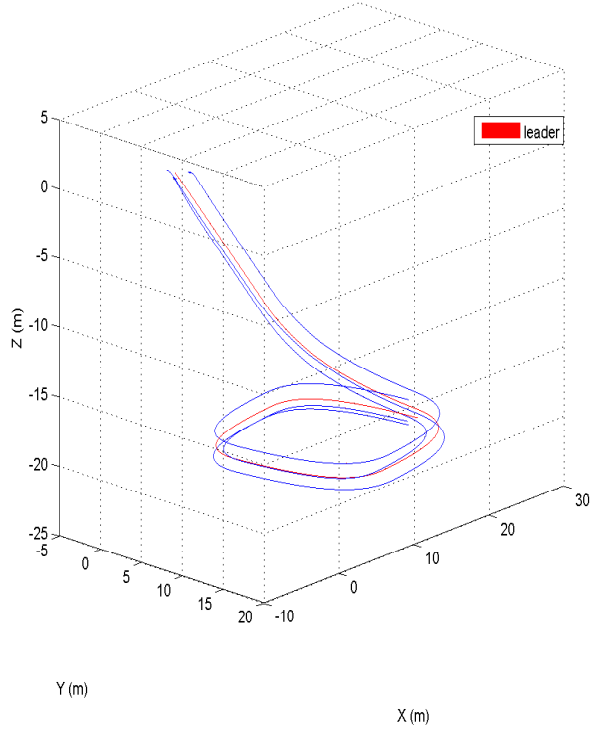


Figure 5.14: 3D trajectory tracking of the leader-follower without uncertainty.

By applying  $\bar{x}$  analysis on the data error of the formation of UVMS without uncertainty in parameters, the table below is obtained :

Error	leader and follower1	leader and follower2	leader and follower3
<i>Minimum</i>	0.000	0.056	0.000
<i>Maximum</i>	0.645	0.659	0.681
<i>Mean</i>	0.347	0.418	0.453
<i>Variance</i>	0.036	0.014	0.020
<i>Stand dev</i>	0.190	0.118	0.143

Table 5.5: The  $\bar{x}$  analysis for the formation of UVMS without uncertainty in parameters

### 5.2.1.2 Formation of UVMs with uncertainty in parameters

The uncertainty in parameters (water density and the inertia matrix for the UVMs) was not considered in the figures from figure (5.11) to figure (5.14), but now it is considered in six scenarios. The first scenario is (no error in water density and 25% constant error equal to scale in inertia matrix), the second scenario is (no error in water density and 100% constant error equal to scale in inertia matrix), the third scenario is (no error in water density and 50% vary error equal to sine wave in inertia matrix), the fourth scenario is (no error in water density and 100% vary error equal to sine wave in inertia matrix), the fifth scenario is (75% constant error in water density and 100% vary error equal to sine wave in inertia matrix), and the last scenario is (100% constant error in water density and 100% vary error equal to sine wave in inertia matrix) .

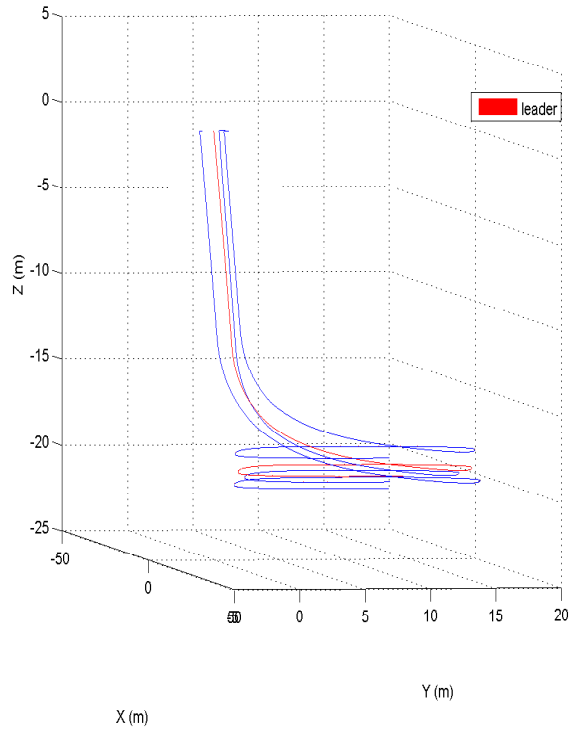


Figure 5.15: 3D trajectory tracking of the leader-follower with 25% constant uncertainty in inertia matrix.

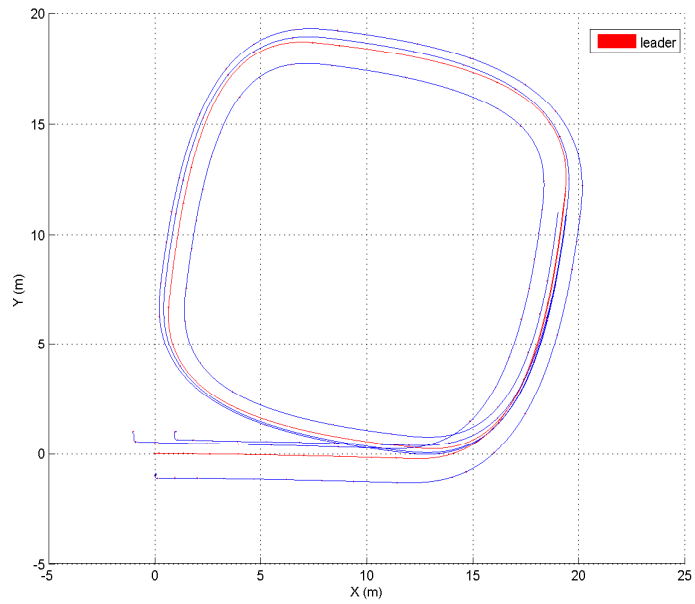


Figure 5.16: X-Y Semi Circle trajectory tracking of the leader-follower with 25% constant uncertainty in inertia matrix.

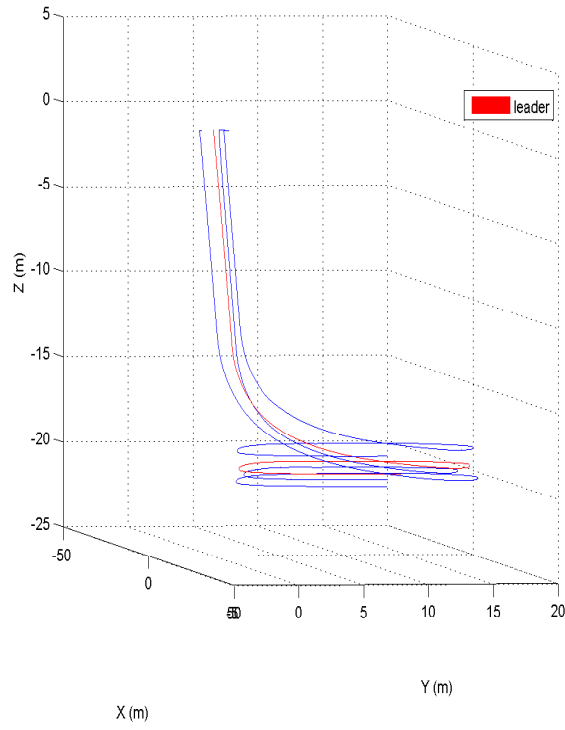


Figure 5.17: 3D trajectory tracking of the leader-follower with 100% constant uncertainty in inertia matrix.

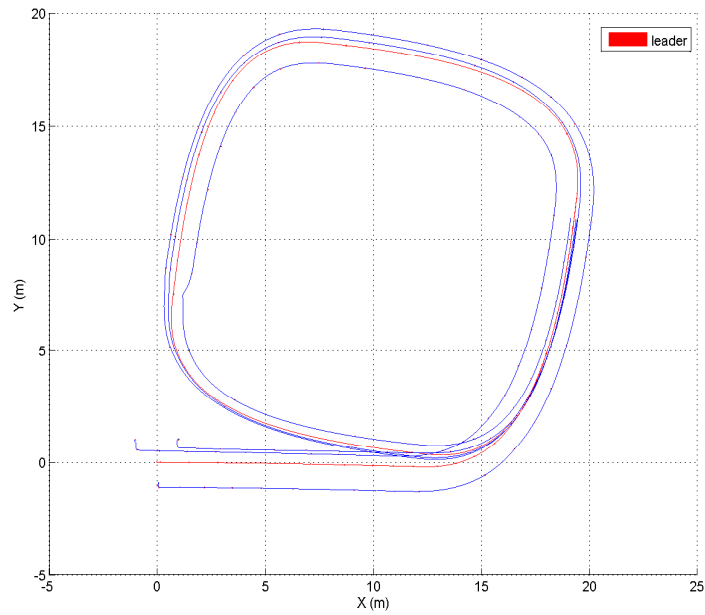


Figure 5.18: X-Y Semi Circle trajectory tracking of the leader-follower with 100% constant uncertainty in inertia matrix.



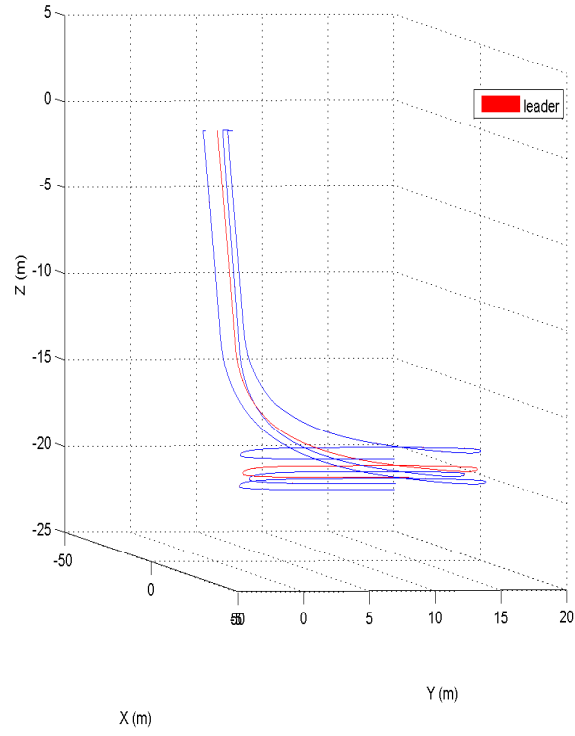


Figure 5.19: 3D trajectory tracking of the leader-follower with 50% vary uncertainty in inertia matrix.

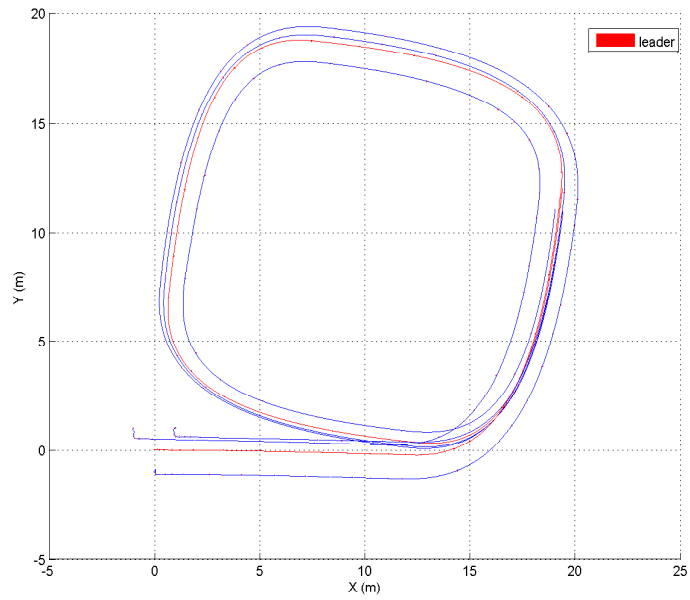


Figure 5.20: X-Y Semi Circle trajectory tracking of the leader-follower with 50% vary uncertainty in inertia matrix.

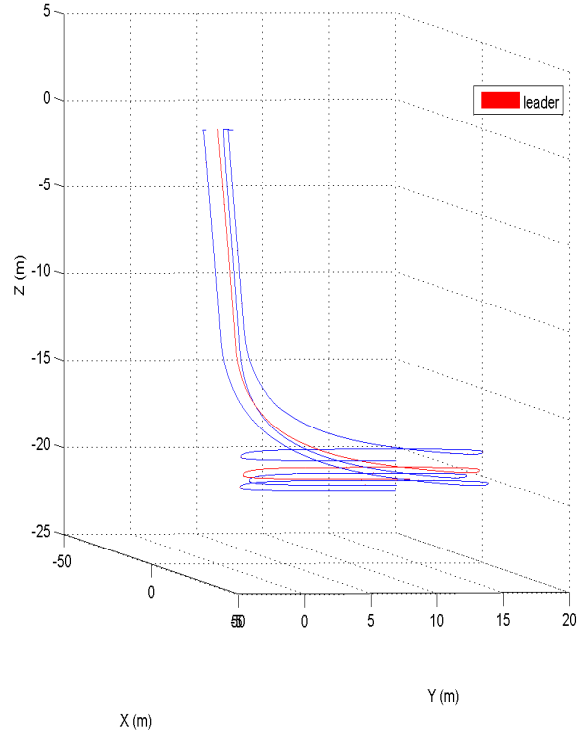


Figure 5.21: 3D trajectory tracking of the leader-follower with 100% vary uncertainty in inertia matrix.

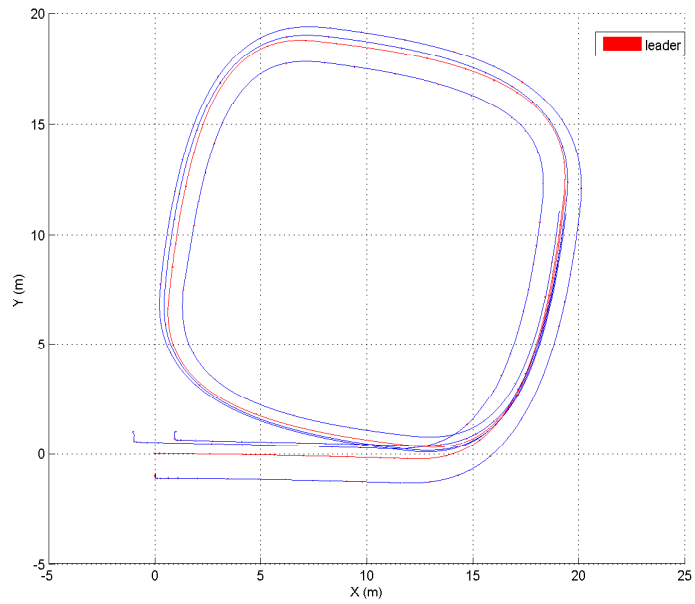


Figure 5.22: X-Y Semi Circle trajectory tracking of the leader-follower with 100% vary uncertainty in inertia matrix.

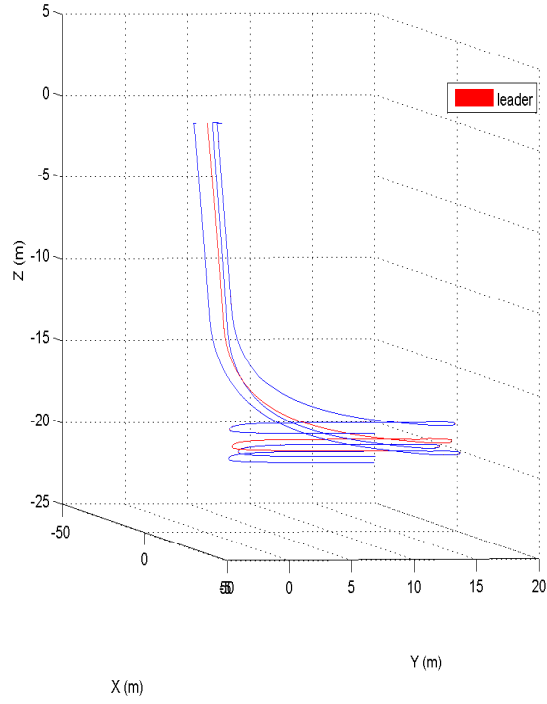


Figure 5.23: the leader-follower with 100% vary uncertainty in inertia matrix and 75% constant uncertainty in water density.

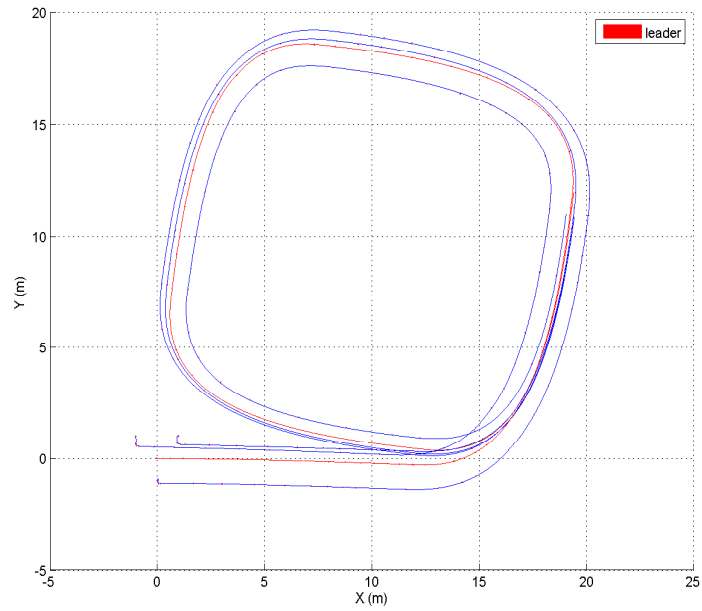


Figure 5.24: X-Y of the leader-follower with 100% vary uncertainty in inertia matrix and 75% constant uncertainty in water density.

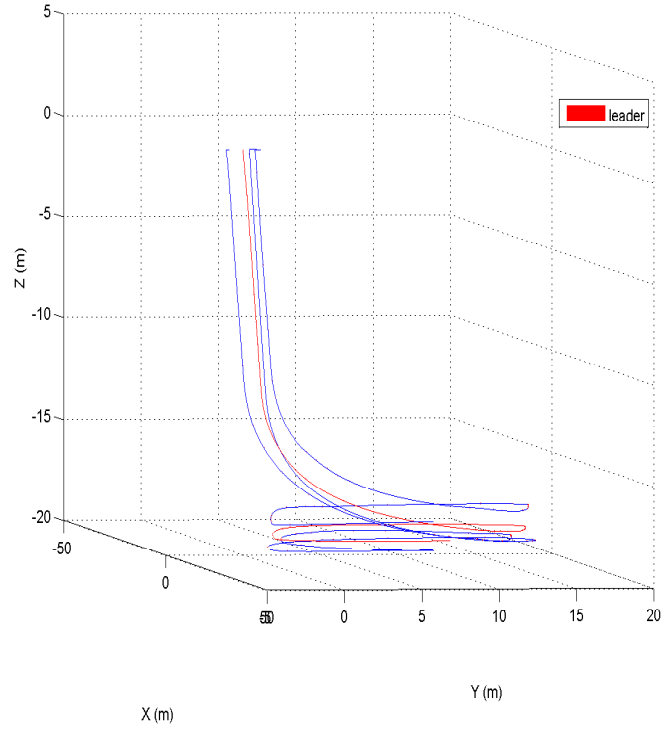


Figure 5.25: the leader-follower with 100% vary uncertainty in inertia matrix and 100% constant uncertainty in water density.

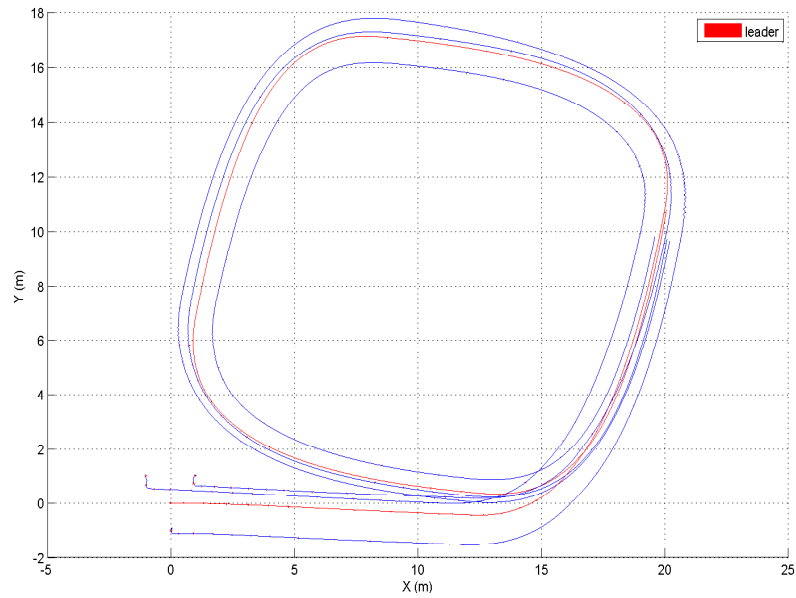


Figure 5.26: X-Y of the leader-follower with 100% vary uncertainty in inertia matrix and 100% constant uncertainty in water density.

By applying  $\bar{x}$  analysis on the data error of the formation of UVMs with uncertainty in parameters for last case, the table below is obtained :

Error	leader and follower1	leader and follower2	leader and follower3
<i>Minimum</i>	0.000	0.000	0.000
<i>Maximum</i>	1.961	1.965	0.792
<i>Mean</i>	0.469	0.529	0.436
<i>Variance</i>	0.157	0.135	0.053
<i>Stand dev</i>	0.396	0.367	0.230

Table 5.6: The  $\bar{x}$  analysis for the formation of UVMs with uncertainty in parameters

## 5.2.2 Cooperating task

In this case the vehicle and the manipulator are doing task, and this also considered with/without uncertainty in parameters.

### 5.2.2.1 Formation of UVMs without uncertainty in parameters

The figures (5.27) and (5.28) are shown how the group of three UVMs can follow their leader in 3D space and the figure (5.29) is shown the formation of the manipulators of the group of three UVMs with their leader manipulator in 3D space .

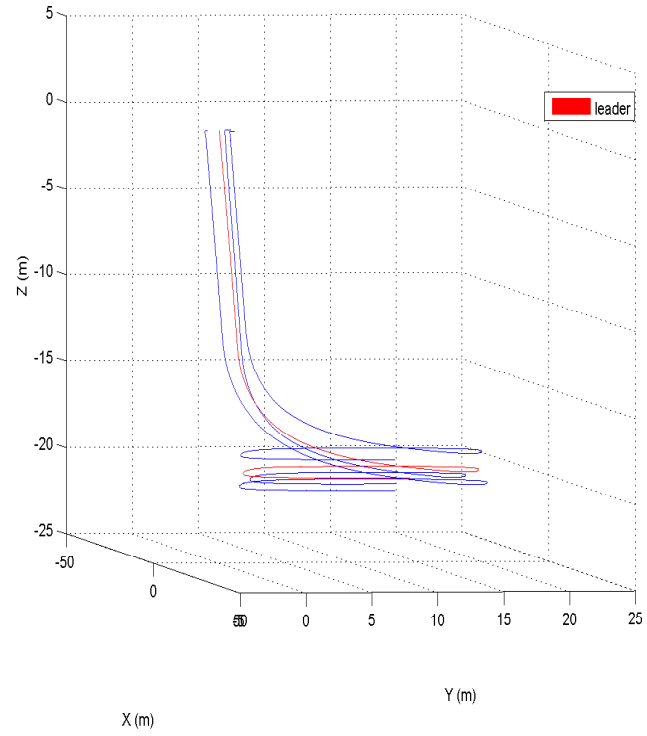


Figure 5.27: 3D trajectory tracking of the leader-follower without uncertainty.

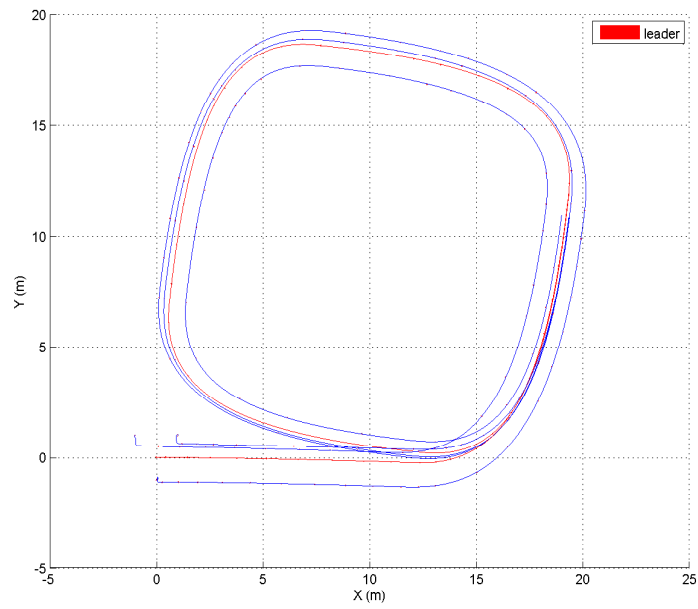


Figure 5.28: X-Y Semi Circle trajectory tracking of the leader-follower without uncertainty in parameters.

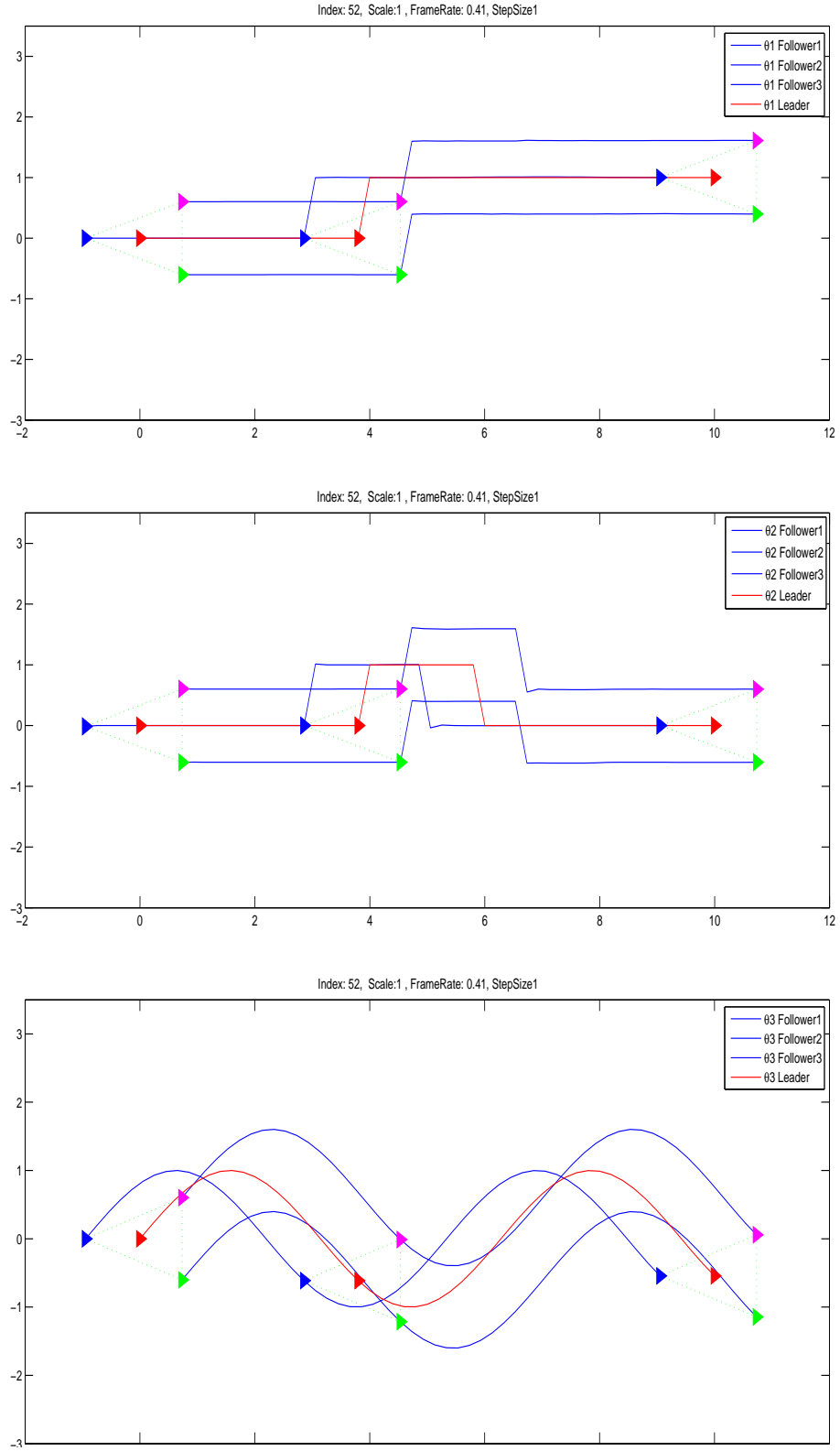


Figure 5.29: The formation of manipulator angles trajectory tracking without uncertainty.

By applying  $\bar{x}$  analysis on the data error of the formation of UVMs without uncertainty in parameters, the table below is obtained :

Error	leader and follower1	leader and follower2	leader and follower3
<i>Minimum</i>	0.000	0.056	0.000
<i>Maximum</i>	0.645	0.659	0.681
<i>Mean</i>	0.347	0.418	0.453
<i>Variance</i>	0.036	0.014	0.020
<i>Stand dev</i>	0.190	0.118	0.143

Table 5.7: The  $\bar{x}$  analysis for the formation of UVMs without uncertainty in parameters

### 5.2.2.2 Formation of UVMs with uncertainty in parameters

The uncertainty in parameters (water density and the inertia matrix for the UVMs) was not considered in the figures (5.27) and (5.29), but now it is considered. The figures (5.30) and figure (5.31) are shown how the group of three UVMs can follow their leader in 3D space and the figure (5.32) is shown the formation of the manipulators of the group of three UVMs with their leader manipulator in 3D space .



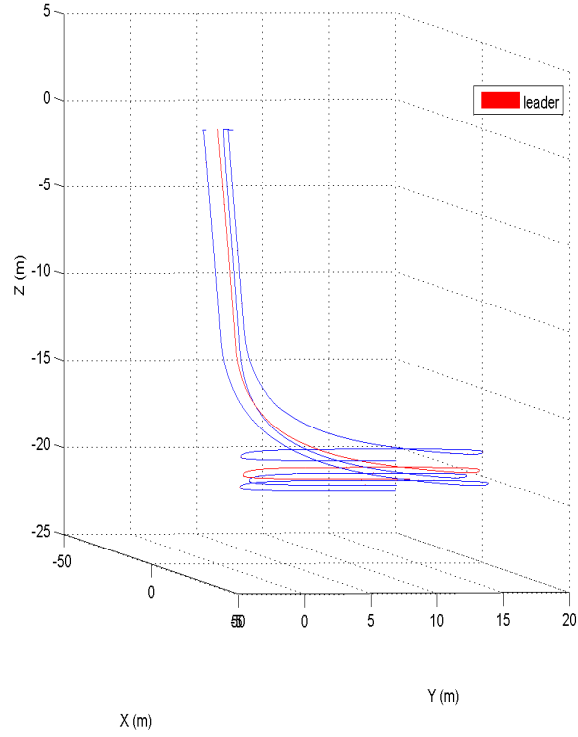


Figure 5.30: 3D trajectory tracking of the leader-follower with 100% vary uncertainty in inertia matrix and 100% vary uncertainty in water density.

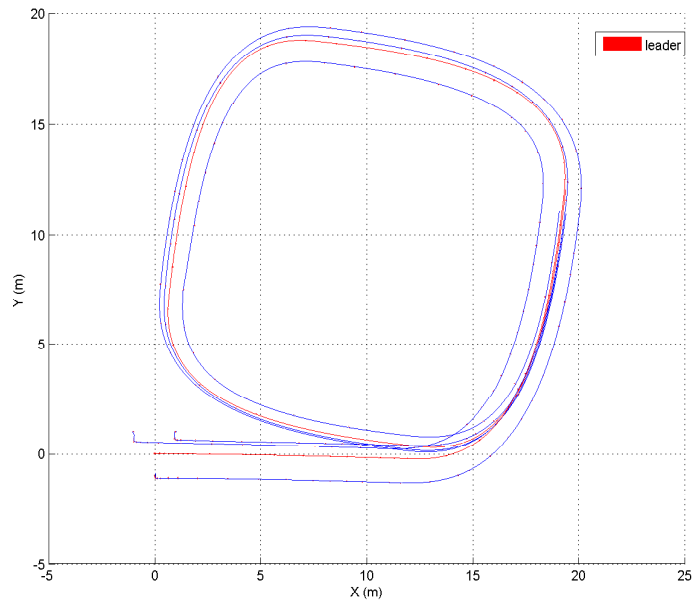


Figure 5.31: X-Y Semi Circle trajectory tracking of the leader-follower with 100% vary uncertainty in inertia matrix and 100% vary uncertainty in water density.

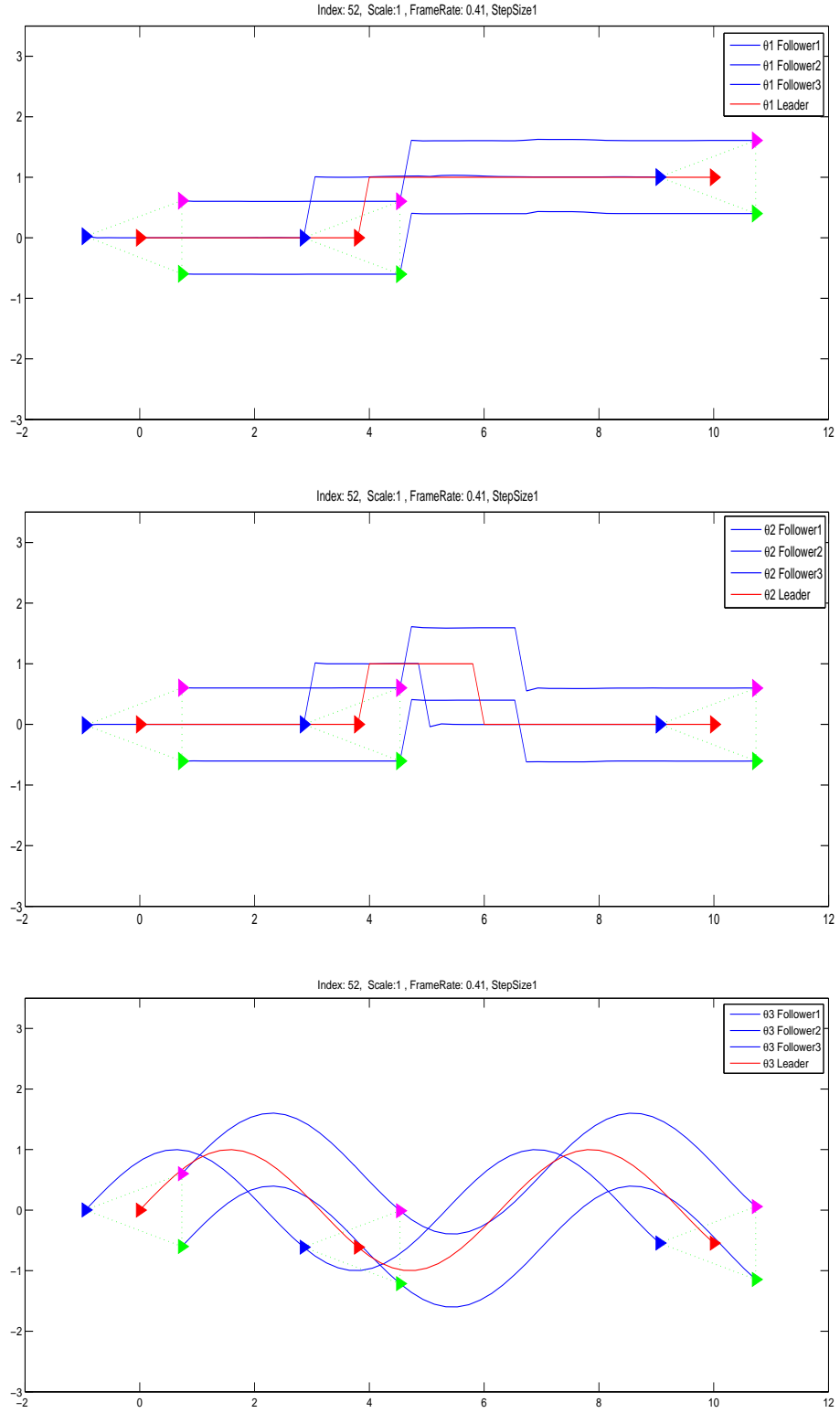


Figure 5.32: The formation of manipulator angles trajectory tracking with uncertainty in parameters.

By applying  $\bar{x}$  analysis on the data error of the formation of UVMs with uncertainty in parameters , the table below is obtained :

Error	leader and follower1	leader and follower2	leader and follower3
<i>Minimum</i>	0.000	0.000	0.000
<i>Maximum</i>	1.961	1.965	0.792
<i>Mean</i>	0.469	0.529	0.436
<i>Variance</i>	0.157	0.135	0.053
<i>Stand dev</i>	0.396	0.367	0.230

Table 5.8: The  $\bar{x}$  analysis for the formation of UVMs with uncertainty in parameters

## 5.3 Results of leader-follower containment in 2D

In the results of leader-follower containment two different cases have been considered:

### 5.3.1 Vehicle doing task

In this case the manipulator remained fixed while the vehicle doing task, and this considered with/without uncertainty in parameters.

The leader-follower containment is shown in the figures below. The figures from figure (5.33) to figure (5.38) are shown how the group of nine UVMs can localize themselves inside four leaders in 2D space. The map of a fleet of nine

agents along sine shape with their leaders under containment control based on L1 Adaptive control, it is shown in figure (5.39).

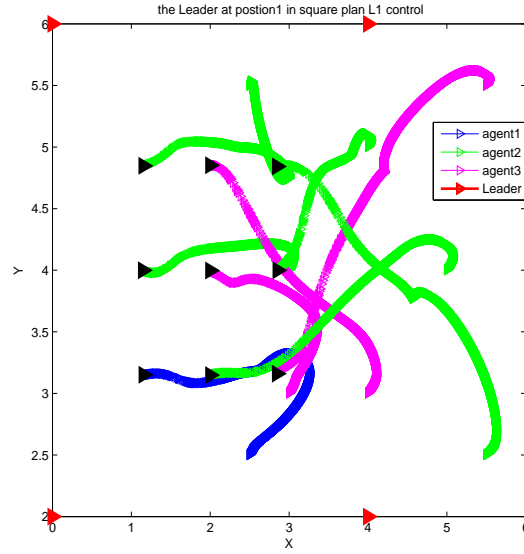


Figure 5.33: Leader1,Leader2,Leader3,Leader4 at  $(0,2),(0,6),(4,2),(4,6)$  and followers at random positions.

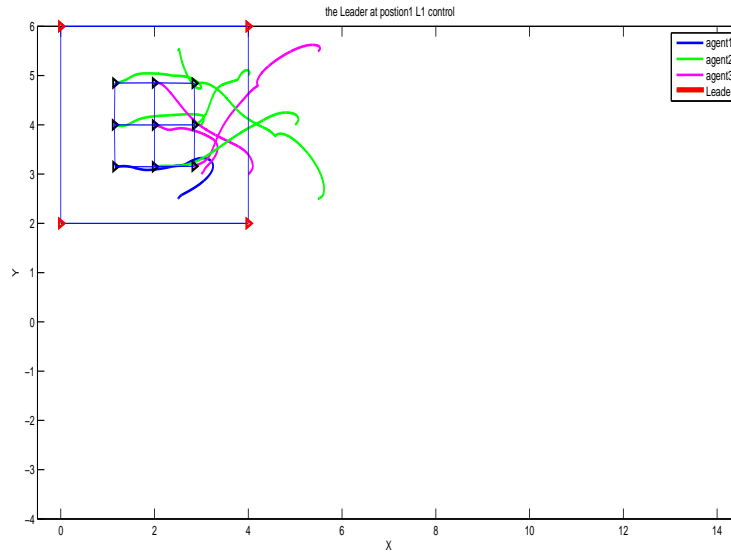


Figure 5.34: Leader1,Leader2,Leader3,Leader4 at  $(0,2),(0,6),(4,2),(4,6)$  and followers at random positions.

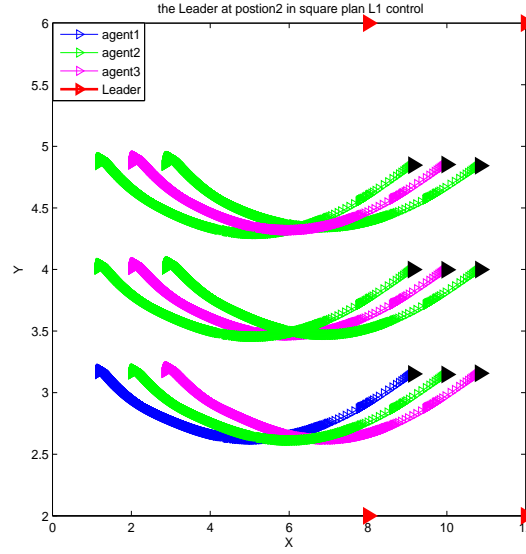


Figure 5.35: Leader1,Leader2,Leader3,Leader4 at  $(8,2),(8,6),(12,2),(12,6)$  and followers at final positions in figure 5.33.

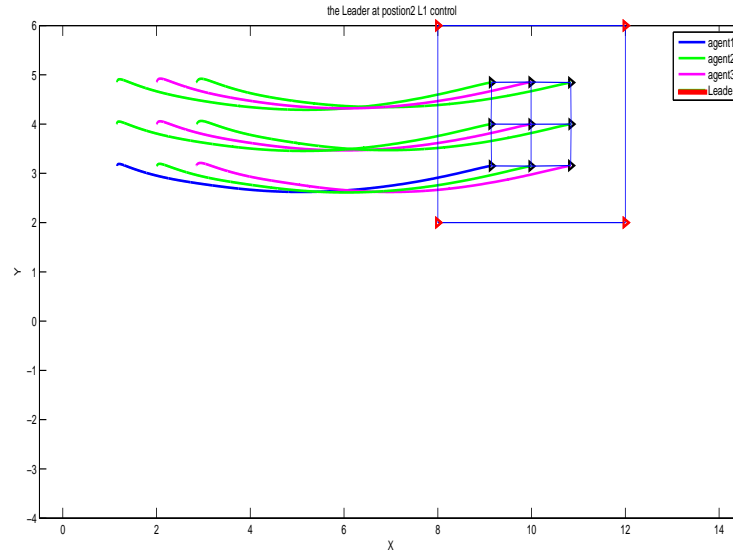


Figure 5.36: Leader1,Leader2,Leader3,Leader4 at  $(8,2),(8,6),(12,2),(12,6)$  and followers at final positions in figure 5.33.

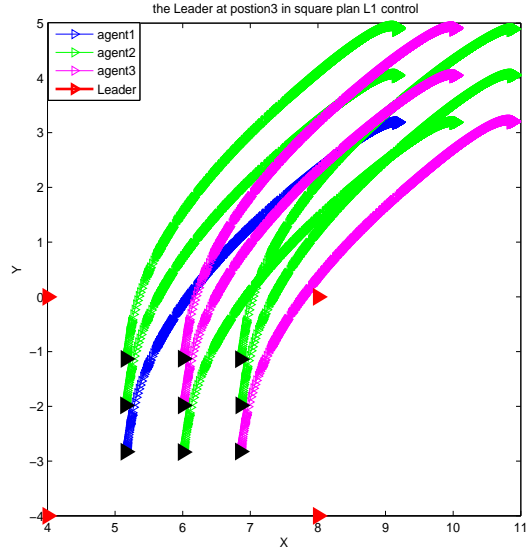


Figure 5.37: Leader1,Leader2,Leader3,Leader4 at  $(4,-4),(4,0),(8,-4),(8,0)$  and followers at final positions in figure 5.35.

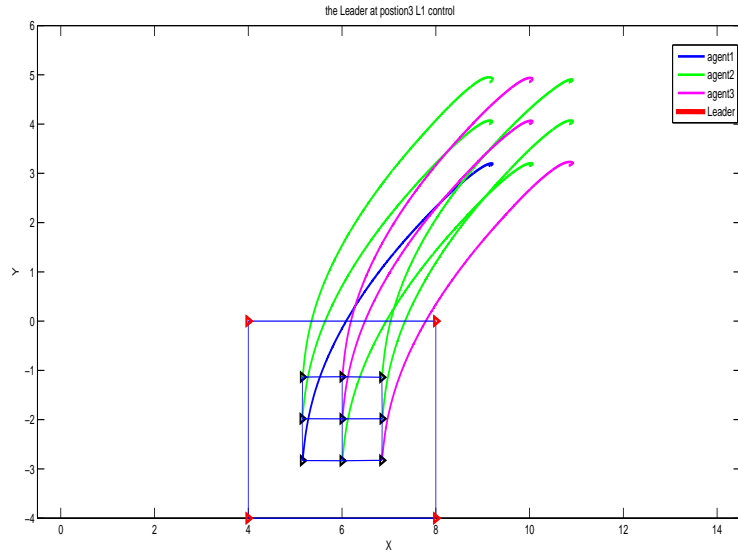


Figure 5.38: Leader1,Leader2,Leader3,Leader4 at  $(4,-4),(4,0),(8,-4),(8,0)$  and followers at final positions in figure 5.35.

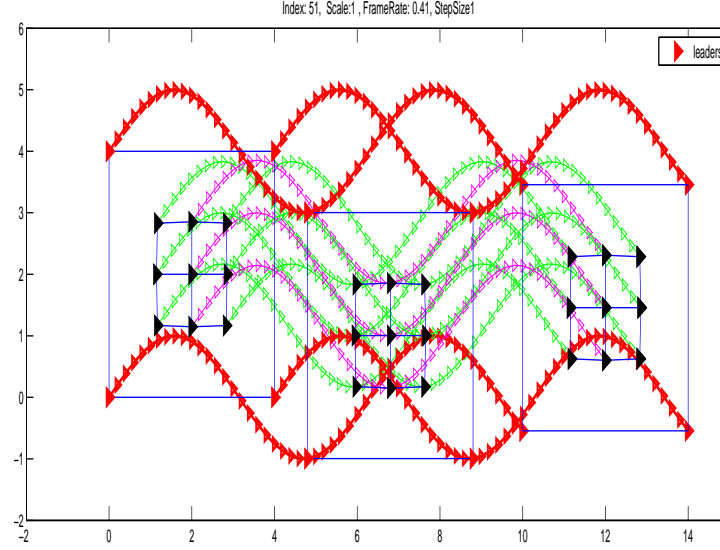


Figure 5.39: The map of a fleet of nine agents along sine shape . with their leaders under containment control based on  $L_1$  Adaptive control .

### 5.3.2 Cooperating task

In this case the vehicle and the manipulator are doing task, and this also considered with/without uncertainty in parameters.

#### 5.3.2.1 Containment of UVMs without uncertainty in parameters

The figures (5.40) is shown how the group of nine UVMs can follow their leaders in 2D space and the figure (5.41) is shown the containment of the manipulators of the group of nine UVMs with their leaders manipulator in 2D space .

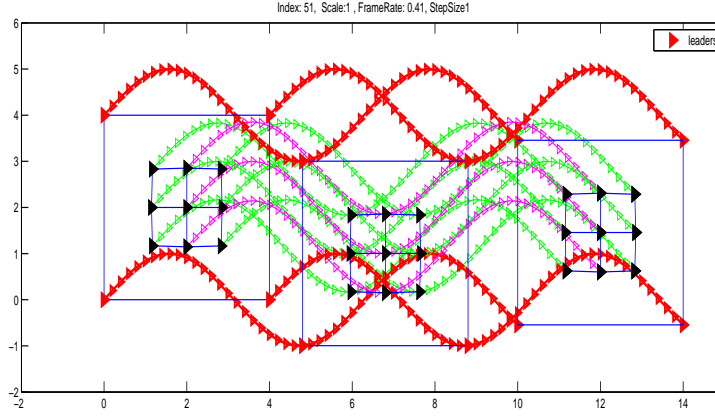


Figure 5.40: The map of a fleet of nine agents along sine shape . with their leaders under containment control without uncertainty in parameters.

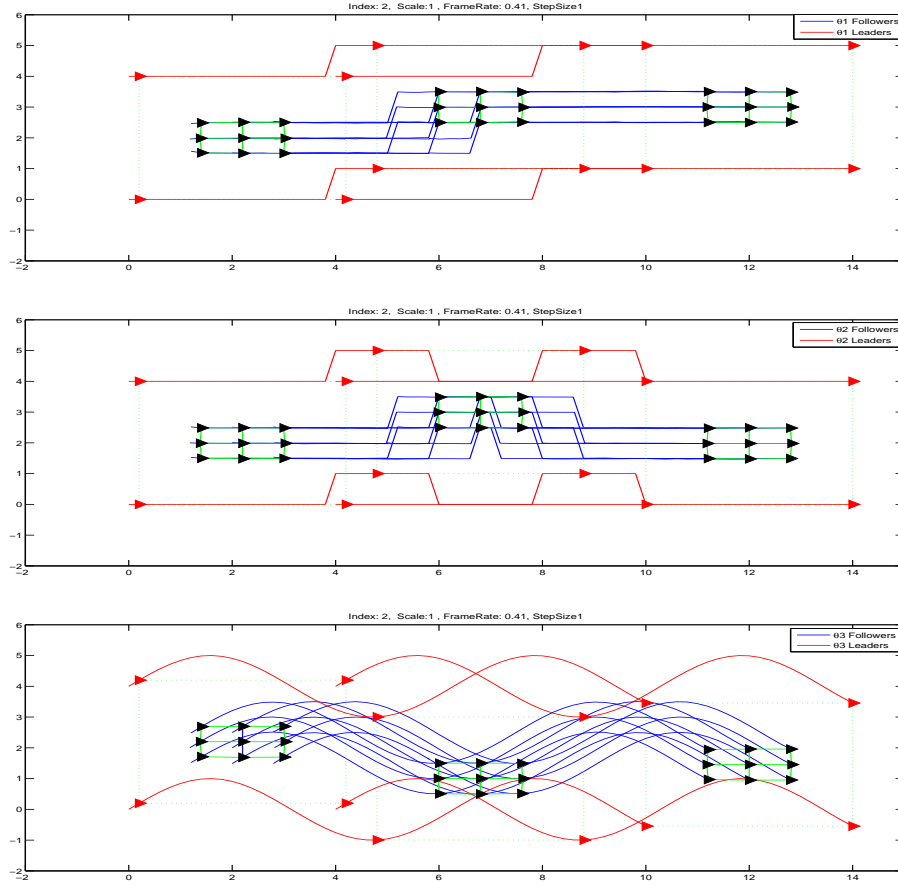


Figure 5.41: The containment of manipulator angles trajectory tracking without uncertainty in parameters.



### 5.3.2.2 Containment of UVMs with uncertainty in parameters

The uncertainty in parameters (water density and the inertia matrix for the UVMs) was not considered in the figures (5.40) and (5.41), but now it is considered. The figure (5.42) is shown how the group of nine UVMs can follow their leaders in 2D space and the figure (5.43) is shown the containment of the manipulators of the group of nine UVMs with their leaders manipulator in 2D space .

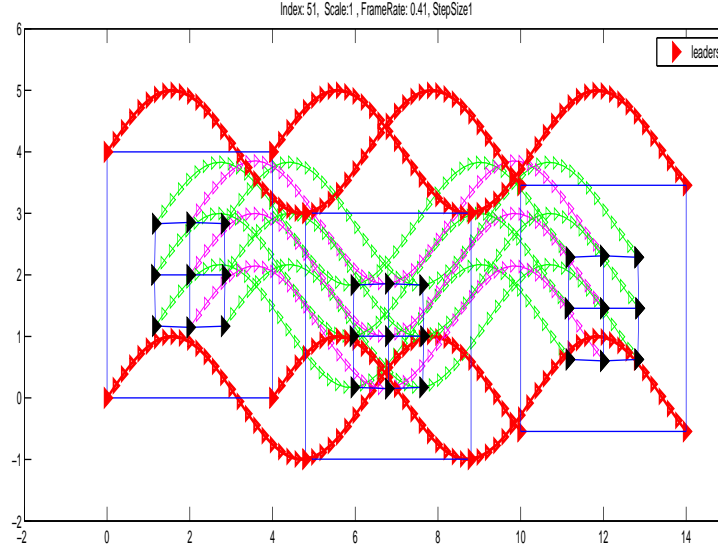


Figure 5.42: The map of a fleet of nine agents along sine shape . with their leaders under containment control with uncertainty in parameters .

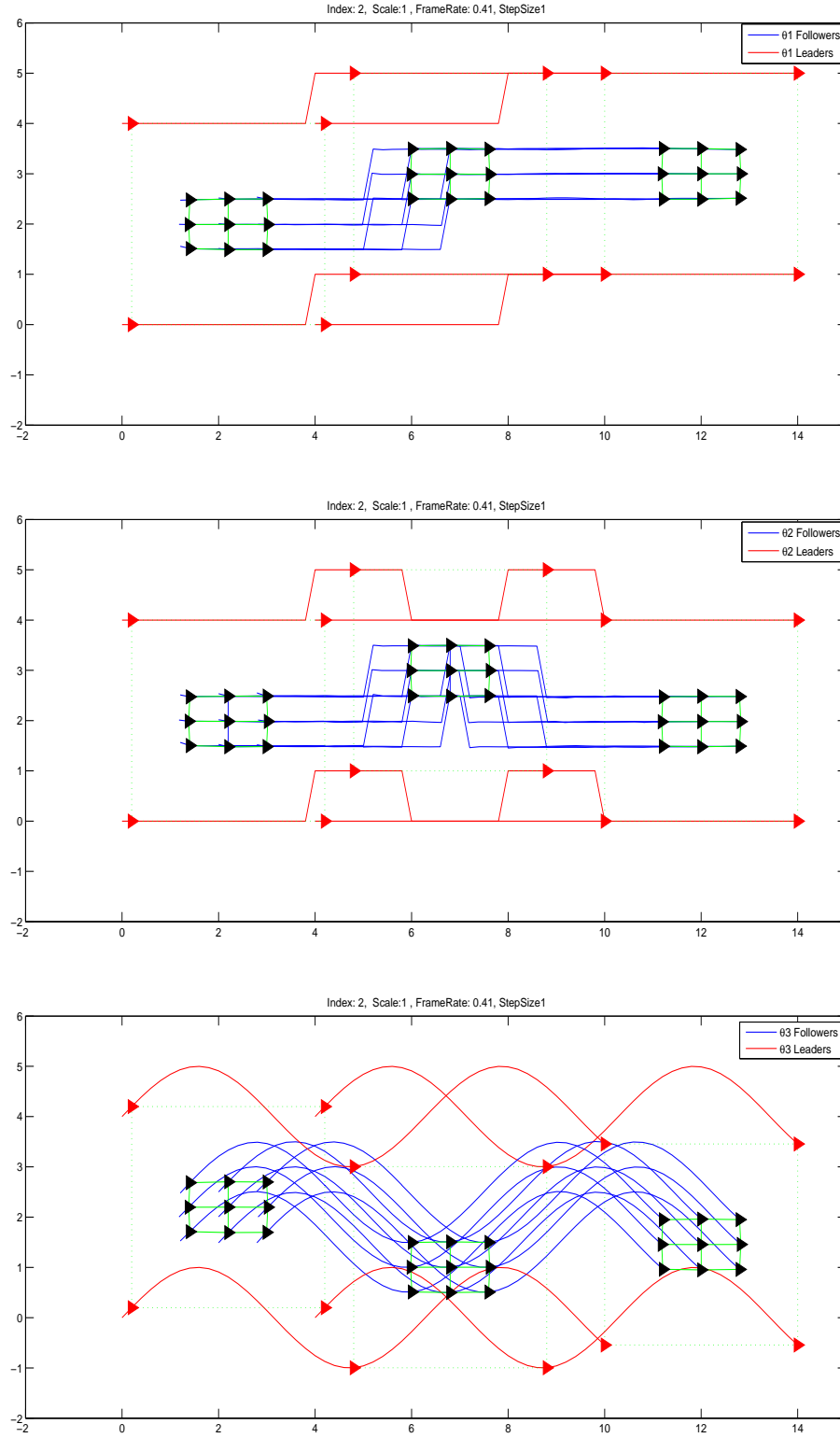


Figure 5.43: The containment of manipulator angles trajectory tracking with uncertainty in parameters.

## CHAPTER 6

# CONCLUSION AND FUTURE WORK

In this thesis, a new framework for leader-follower formation control of a fleet of UVMs is presented. The formation control of fleet navigation is developed based on  $\mathcal{L}_1$  adaptive control and artificial potential field formation strategy. The  $\mathcal{L}_1$  adaptive control is used to control dynamic model of the UVMs, and the attractive and repulsive potential fields are used to control UVMs positions and hold them to their desired paths with respect to their leader. In case of disturbances and uncertainty in the dynamics, the  $\mathcal{L}_1$  adaptive controller showed better performance than the other controllers in literature. Also the designed framework can, not only, be used for a group of three UVMs, but can also be used effectively for containment control of UVMs where nine followers can localize themselves inside four leaders. Simulation results prove that the  $\mathcal{L}_1$  adaptive controller increased performance in terms of fast and robust adaptation. The formation stability has been presented

based on Lyapunov analysis.

## **6.1 Future work**

The new framework presented in this thesis only applies to the fully-actuated UVMS. This framework would be extended to the under-actuated UVMS. In this case, the control will be more complicated because the size of the degree of freedom will be increased. The environment in the ocean is an unknown environment, and it contains both static and moving obstacles. This framework can be extended to the obstacles avoidance in future work.

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# Vitae

- Name: Siddig Mustafa Elkhider Elnaiem
- Nationality: Sudanese
- Date of Birth: 27/10/1985
- Email: *siddig-ce1@yahoo.com*
- Permenant Address: Systems Engineering Dept.- KFUPM University - Dhahran  
- KSA
- Bsc degree from Algazira University (Sudan) in Computer Engineering (2009)
- Ms degree from KFUPM (KSA) in Systems Engineering-Control Systems  
(2014)